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The frontier of Social Impact Finance: Theory and two case studies*

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Abstract

Social impact bonds (SIBs) are a novel and innovative form of public-private partnership financing social services performed by a best-practice selected non-governmental third entity. In our paper we outline a SIB theoretical model identifying government and private investors' participation constraints and we discuss the conflicts of interests that may arise among the different actors involved in presence of asymmetric information. We apply our theoretical model to two investment cases concerning contrast to jail recidivism and health budget project. We show conditions for viability of the SIB scheme in both cases under reasonable parametric conditions, provide sensitivity analysis on crucial parameters, and calculate participants' payoffs under different assumptions.

JEL numbers: G23 (financial instruments); I31 (General welfare, wellbeing).

Keywords: social impact finance, jail recidivism, health budget.

1 Introduction

Social impact bonds (SIBs) are innovative financial instruments that have recently drawn increasing attention from policymakers and researchers. They are financial mechanisms aimed at attracting private capital to finance the provision of a social service from a high-qualified organisation, which is expected to reduce costs for the commissioners (usually public administrations) in service delivery. SIBs are also called pay-for-success or pay-for-performance bonds (OECD, 2016; Gustafsson-Wright et al., 2015) as they incorporate a bet: if the performance of the social service provider improves what agreed with the counterparts, some costs saved by the commissioner become profits for the private investors financing the initiative.

The structure of SIBs is complex and involves several actors: a private financial intermediary issuing the bonds, private investors buying the bonds, the commissioner (most often a government or a local administration), the social service provider, the beneficiaries of the social service, and the independent validator who ascertains whether the project impact due to the service provider improves an ex ante agreed level of performance.

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Given the above mentioned features, SIBs show at least two potentials. First, SIBs leverage private financial resources to invest in the provision of social services by transferring the risk usually run by the commissioner to private investors. Second, SIBs promote a culture of quality and innovation in social service provision. In fact, the SIB scheme provides the financial intermediary with a strong incentive to select the highest quality provider and ensure the service success. This implies that successful SIBs may be win-win operations as they can create savings for the government budget and, at the same time, increase the quality of public services (OECD, 2016).

In spite of this promising potential, some caveats related to the articulated SIB structure cannot be neglected. First, SIBs can work for a limited number of social activities. These are activities where the commissioner has a positive economic benefit which, together with the service provider performance, is clearly identifiable and measurable to be agreed upon by the counterparts. For instance, SIBs have been so far issued for interventions to combat jail recidivism[1] or school abandonment, where costs and outcomes for the public commissioner are observable and measurable. However, SIBs can also find applications in job training, health care and prevention campaigns, provision of disability services, and foster care (OECD, 2016). In principle, the boundaries of SIBs viability can be extended if we consider the role of responsible investors willing to pay for a social premium and activities which do not produce direct economic benefits for the commissioner but where a conventional economic value measuring its willingness to pay can be applied.[2] A second SIB limitation is represented by time delays and high costs of project impact evaluations, especially when the most advanced methodologies such as randomised control trials are applied. The third caveat arises when the project selection is only driven by performance, as this would exclude the most challenging cases and select the most likely to achieve the outcome ('cherry-picking' effect), the highest achievers ('cream-skimming' effect), or the easiest to reach ('parking' effect) (OECD, 2016). This limitation can be overcome as far as the SIB is not the only exclusive financial tool providing the service, even though an upward bias in estimating project benefit would remain. Last but not least, SIBs require articulated contracts to reduce conflicts of interest among the number of actors involved (see section 5) and, as a consequence, they may have high transaction costs.

Our paper provides an original contribution to the newborn SIB literature by outlining theoretical features and discussing two applications. First, we outline an 'impossibility theorem' that states that there exists no feasible SIB satisfying both the government and the private investor participation constraints if the government is risk averse or risk neutral and the activity investment costs are assumed to be the same for the government and the private investors. In a second scenario, we relax this last assumption and show that, when bureaucratic costs or different efficiency in the public sector make the activity investment costs higher than those of the private sector, a SIB is feasible even with a risk neutral or risk averse government.

The paper is divided into six sections including introduction and conclusions. In the second section we sketch the perfect information SIB model outlining participation constraints for the government and private financiers. The third section discusses potential conflicts of interest among SIB actors when removing the assumption of perfect information. In the fourth section we apply the SIB scheme to a project reducing jail recidivism, and in the fifth section we apply it to a health budget project. The sixth section concludes.

[1] Three of the most interesting SIB projects around the world have been in this field. The Petersborough project in the UK (Disley et al., 2011; Disley and Rubin, 2014), the Rikert Island project in the US, and the Juvenile Justice Pay for Success Initiative, Massachussets, US.

[2] An example of such an activity is provided by job creation where the implicit willingness to pay is measured by government expenditure in active labour policies per estimated number of jobs created with those policies.

2 How SIBs work

There are three ways in which the government can raise private funds. First, the government issues a financial asset as a fraction of the total investment with each share having the same risk-return characteristics of the aggregate financial investment. Second, private investors are concentrated in one large financial intermediary 'buying' the equivalent asset – this is the case of the intermediated SIBs, according to the taxonomy of Goodall (2014) and OECD (2016). Third, a large financial intermediary chooses an 'originate to distribute' model, that is, it creates a special purpose vehicle issuing a financial asset sold to market investors with a fixed interest dividend in case of success and no dividend in case of failure. In this case, the large financial intermediary covers costs and gets its margin soon, and distributes the risk on small private investors. For the sake of simplicity, in our theoretical model we refer to the second approach even though also the other two can be reconciled with the model under reasonable assumptions.

3 The perfect information SIB Model

Consider a social activity which requires an investment X that may lead to a success outcome Y with probability $p \in [0, 1]$ or a failure outcome F with probability $1 - p$. The investment in the social activity has an expected return $pY + (1 - p)F$ and is risky since we assume $F < X < Y$. The government is the commissioner of the activity and decides whether directly performing the activity or delegating it to a third agent through a SIB scheme. To mimic the characteristics of the existing SIBs, we assume that the government raises private capital to cover the investment cost X, creates a guarantee fund $\phi \in (0, 1)$ as a share of the investment X in order to reduce the risk of the private investor, and agrees to share a portion $\pi \in (0, 1)$ of the success outcome with the private investor.

Thus, a SIB contract is characterised by a pair of (ϕ, π) that satisfies government and private investor participation constraints. In the next section we describe the optimal SIB, that is the SIB that maximises government expected utilities without charging additional costs to the private investor. Note that in our model the agency implementing the activity (e.g., a non-profit organisation), the intermediary agents, and the external evaluator do not play any role. This is possible if we assume that the costs the government would incur in case of direct implementation equal the costs that the private investor faces with a SIB. We relax this assumption in section 3.1 where we assume different costs for the government and the private investors.

Government participation constraint

To contract a SIB with private investors, a government requires expected revenues from the SIB being greater than or equal to those arising from direct implementation.
Hence, the government expected gain in case of direct financing of the social activity is

$$U_{G_D} = p(Y - X) + (1 - p)(F - X)$$

while, under the SIB scheme, it writes

$$U_{G_{SIB}} = p(1 - \pi)(Y - X) - (1 - p)\phi X$$

In essence, the SIB allows the government and private investors to trade part of the risk with a share of profits. We also assume that the government may have different risk attitudes. In particular, the government may be willing to accept an extra loss (i.e., risk seeking), pretend an extra gain (i.e., risk averse), or not require anything additional (i.e., risk neutral) from the SIB.

Thus, the commissioner participation constraint holds if the expected benefit from the SIB is greater than or equal to that from direct implementation, that is

$$\begin{aligned} U_{G_D} &\leq U_{G_{SIB}} + k \\ p\pi(Y-X) + (1-p)\big(F-(1-\phi)X\big) &\leq k. \end{aligned} \tag{Gc}$$

where k represents the risk attitude of the government and can be positive (i.e., the government is risk seeking), negative (i.e., the government is risk averse), or equal to 0 (i.e., the government is risk neutral).

Equation (Gc) and its partial derivatives[3] show that the government incentive to participate lowers with a higher private investors' share π, guarantee fund share ϕ, success outcome Y, and failure outcome F, while it increases with a higher investment cost X as long as $p\pi \geq (1-p)\phi$, coeteris paribus. As for the probability, a higher probability of success decreases the government incentive to participate if and only if $\pi(Y-X) - (F-(1-\phi)X) > 0$, that is if and only if the government faces higher loss in case of success than gain in case of failure.

Private investors participation constraint

From the private investor's perspective, the social activity corresponds to an equivalent asset r with mean

$$\mathbf{E}[r] = p\pi\frac{Y-X}{X} + (1-p)\frac{F-X+X\phi}{X}$$

and standard deviation[4]

$$\sigma^2(r) =: \sigma^2(\pi,\phi) = p(1-p)\frac{1}{X^2}\big(\pi(Y-X) - F + X - X\phi\big)^2. \tag{1}$$

where by abuse of notation we also write $\sigma^2(\pi,\phi)$ to highlight the dependent variables affecting the variance.

The private sector participation constraint is met if the equivalent asset lies above or along the efficient frontier (EF) (or security market line) represented by

$$\mathbf{E}_{EF}[r] \geq a_0 + a_1\sigma^2(r) \tag{Pc}$$

that is

$$p\pi(Y-X) + (1-p)(F-X+X\phi) \geq X\Big(a_0 + a_1\frac{p(1-p)}{X^2}\big(\pi(Y-X) - F + X - X\phi\big)^2\Big)$$

where the intercept a_0 and slope a_1 represent the risk-free interest rate and the risk premium respectively, and they can be estimated using historical nominal rates of return and standard deviations of standard assets such as stocks and bonds. In other words, private investors participate to the venture only if the SIB returns as an equivalent asset with mean and standard deviations that are competitive in financial markets and do not lie below the security market line (Pc).

[3]The partial derivatives with respect to each variable write $\frac{\partial\left(p\pi(Y-X)+(1-p)\left(F-(1-\phi)X\right)\right)}{\partial\pi} > 0$, $\frac{\partial\left(p\pi(Y-X)+(1-p)\left(F-(1-\phi)X\right)\right)}{\partial\phi} > 0$, $\frac{\partial\left(p\pi(Y-X)+(1-p)\left(F-(1-\phi)X\right)\right)}{\partial Y} > 0$, $\frac{\partial\left(p\pi(Y-X)+(1-p)\left(F-(1-\phi)X\right)\right)}{\partial X} < 0 \iff p\pi > (1-p)\phi$, and $\frac{\partial\left(p\pi(Y-X)+(1-p)\left(F-(1-\phi)X\right)\right)}{\partial F} > 0$

[4]Proof in Appendix A.

Given the above mentioned SIB characteristics the 'expected public expenditure multiplier' (i.e., the ratio between the expected value of public expenditure revenues and the expected cost of government participation in the SIB) is given by

$$m_{SIB} := \frac{p(Y-X)(1-\pi)}{(1-p)\phi X}.$$

The optimal solution

The government maximises its expected gain subject to the participation constraints discussed above. Thus, the maximisation problem writes

$$\begin{array}{lll} & \max_{\pi,\phi} & p(1-\pi)(Y-X)-(1-p)\phi X \\ \text{s.t.} & \text{(Gc):} & p\pi(Y-X)+(1-p)(F-X+X\phi)\leq k \\ & \text{(Pc):} & p\pi(Y-X)+(1-p)(F-X+X\phi)\geq X(a_0+a_1\sigma^2(\pi,\phi)) \end{array}$$

The constraints (Gc) and (Pc) can be jointly satisfied if and only if

$$\begin{aligned} k &\geq X(a_0+a_1\sigma^2(\pi,\phi)) & (2)\\ &> 0, & (3) \end{aligned}$$

where (3) comes from the positive signs of the risk-free interest rate a_0 and the risk premium a_1. Therefore, we have the following impossibility result.

Proposition 1. *Let a social activity require an investment cost X, and lead to a successful output Y with probability p, or a failure outcome with probability $1-p$. There exists no SIB $(\pi,\phi)\neq(0,0)$ implementing the social activity that leaves a non-risk seeking government at least as well as off as it would have been by implementing the social activity without the SIB.*

Proof. If the government is not risk-seeking, then $k\leq 0$, which contradicts the participation constraints $k\geq X(a_0+a_1\sigma^2(\pi,\phi))>0$. □

If the government is risk seeking enough (i.e., $k\geq X(a_0+a_1\sigma^2(\pi,\phi))$, then it would be optimal for it to set its risk coefficient at the minimum, that is $k=X(a_0+a_1\sigma^2(\pi,\phi))$. In this case the government maximisation problem writes

$$\begin{array}{lll} & \max_{\pi,\phi} & p(1-\pi)(Y-X)-(1-p)\phi X \\ \text{s.t.} & \text{(Gc):} & p\pi(Y-X)+(1-p)(F-X+X\phi)= k \\ & \text{(Pc):} & p\pi(Y-X)+(1-p)(F-X+X\phi)\geq X(a_0+a_1\sigma^2(\pi,\phi)) \end{array} \quad (4)$$

and the optimal SIB writes[5]

$$(\pi^*,\phi^*)=(\frac{a_0X}{Y-X},\frac{X-F+Xa_0}{X}) \quad (5)$$

Note that in order to have $\pi^*,\phi^*\geq 0$ and $\pi\leq 1$, we need the following further assumptions:

$$\begin{aligned} \phi^*\geq 0 &\iff F\leq(1+a_0)X & (6)\\ \pi^*\leq 1 &\iff (1+a_0)X\leq Y. & (7) \end{aligned}$$

Equation (6) (respectively, (7)) requires the failure (successful) outcome to be low (high) enough to leave the government (private investors) at least as well as without SIB.

[5]A proof of the solution is shown in Appendix A, where we also show that another solution $(\hat{\pi},\hat{\phi})$ with $\hat{\pi}\geq\pi^*$ may exist.

Comparative statics

The comparative static analysis on the optimal SIB solution shows that higher investment cost implies higher risk for the private financer and therefore a higher X increases both π^* and ϕ^* since $\frac{\partial \pi^*}{\partial X} = \frac{a_0 Y}{(Y-X)^2} > 0$ and $\frac{\partial \phi^*}{\partial X} = \frac{F}{X^2} > 0$. The effect of higher success outcome Y is obviously opposite and therefore it decreases π^* since $\frac{\partial \pi^*}{\partial Y} = -\frac{a_0 X}{(Y-X)^2} < 0$. In the same direction higher failure outcome F is a risk reduction factor for the private investor and, as such, it decreases ϕ^* since $\frac{\partial \phi^*}{\partial F} = -\frac{1}{X} < 0$, that is higher failure outcome. As well a_0 is part of the opportunity cost of investing in the SIB for the private financer and therefore its growth increases both π^* and ϕ^* since $\frac{\partial \pi^*}{\partial a_0} = \frac{X}{Y-X} > 0$ and $\frac{\partial \phi^*}{\partial a_0} = 1 > 0$. Consider finally that the guarantee fund does not depend on Y (as it is reasonable to be since it is used only in case of failure outcome) and the solutions do not depend on a_1 (since the private investor participation constraint is satisfied with equality in the optimal solution) and p. The intuition here is that the probability of success implies both lower risk for the private investor and higher incentive for the government to manage directly the investment without creating a SIB, and the two effects offset each other.

3.1 Governmental and private costs are different

Suppose that the government faces different costs than private investors. In particular, we may assume, on one hand, the investment cost in case the government directly implements the social activity (i.e., X_g) being higher than the same cost private investors would incur with a SIB (i.e., $X_p < X_g$). This can be due to the absence of expertise, the regulation of national contracts increasing costs for the government, or lack of economies of scale. On the other hand, we may also assume that $X_p > X_g$ since a SIB would involve more agents like intermediaries and external evaluators that would have not been involved otherwise. Hence, a more realist scenario assumes $X_g \neq X_p$ with no a priori hypothesis on which agent faces higher costs.

In this case, (2) now writes[6]

$$k \geq X_P(a_0 + a_1\sigma^2(\pi, \phi)) - (X_G - X_P). \tag{8}$$

allowing us to state the following proposition

Proposition 2. *Let a social activity require an investment cost X_g for the government without a SIB, or an investment cost X_p for the private investors with a SIB, and assume that the activity leads to a successful output Y with probability p or a failure outcome with probability $1-p$. In order to have a feasible SIB, we have that*

(i) if $X_g > X_p$, then the government may be risk averse, risk neutral, or risk seeking: More specifically, the government can be risk-averse if and only if $X_G - X_P > X_P(a_0 + a_1\sigma^2(\pi, \phi))$.

(ii) if $X_g < X_p$, then the government must be risk seeking;

Proof. See Appendix A □

With different costs for the government and the private investors, the optimal solution remains as in (5), when it exists. Then, the only condition that changes is the threshold of the risk factor for the government to be incentivised to implement a SIB, that is (8).

[6] See Appendix A for the proof.

4 Two case studies of SIB in action

4.1 *Made in Carcere* and the case of jail recidivism

The Made in Carcere (MiC) project trains inmate women in the craftsmanship sector with the goal of reintegrating them in the job market. The project started in Puglia, Italy, in 2007 where it has been tested for 10 years on a group of 123 women. Trainees produced handcrafted clothes branded as MiC and market discipline helped female inmates to develop job discipline and improve professional and personal skills. The project was particularly successful as it reduced jail recidivism in Lecce, Italy from 70 to 5 percent in 10 years. A SIB scheme may potentially replicate this project at national level therefore involving more beneficiaries and significantly benefiting government budget.

We simulate a hypothetical SIB for a project like MiC in Italy (Table 1). Yearly savings are estimated to be €58,000 per inmate.[7] In our baseline scenario we assume yearly fixed costs equal to 200,000 euros. These costs represent third agent costs, that is payment of resources employed in women training based on real costs of the first MiC project, plus a bonus calculated as 10 percent of revenues for penitentiary policemen. We calculate values for the overall period equivalent asset using a discount rate of 5 percent. We assume that the project succeeds (i.e., it performs as good as in the previous MiC project with recidivism fallen from 70 to 5 percent) with probability 0.8 (good state), and it fails with probability 0.2 (bad state) (reflecting the difficulty to replicate MiC on a larger scale). We conveniently assume that the bad state is represented by a 20 percent loss of the capital invested implying a reduction of recidivism of only 8 percent (i.e., 62 percent of recidivism rate). In addition, we assume on average 3 years of prison for recidivists[8] and that the effect of recidivism reduction produced by the project is uniformly distributed over 10 years (i.e., any year the positive effect of recidivism reduction is produced on 10 percent of beneficiaries). This implies also an assumption of uniform distribution of the remaining years in prison for participants to the project. The 3-year recidivism assumption produces three different revenues for the first year asset, second year asset, and third year asset, since government savings in the third year of the project are three times higher than in the first. Finally, we use $a_0 = 0.434$ as risk-free interest rate and $a_1 = 0.32$ as risk premium as secular references for these two parameters (Siegel, 1992).

This assumptions together with real data estimate the total cost of the project X equal to €3,470,100.00, which in case of success leads to the outcome Y equal to €10,615,276.04, otherwise the outcome F is equal to €1,206,495.51.00. Thus, according to our model in section 3, we have that a SIB for this project is feasible as long as the government is k-risk-averse or the difference between the government and private investors costs exceeds k, with $k = 1,506,023.40$, and in this case it will share 21% of profits (i.e., $\pi = 0.211$) and will guarantee 106% of the

[7] These costs represent the ratio between total government expenditure for the prison system and the number of inmates. Hence they combine variable and fixed costs. As such we are aware that, while variable costs may be related more directly to the number of prisoners, fixed costs may be saved if the reduction is permanent and of a scale allowing to eliminate one prison infrastructure. Given the large number of inmates involved in MiC, it is reasonable to assume that the number is high enough to imply savings also on fixed costs and jail infrastructure. The other implicit assumption requires zero queues, and this is reasonable for some prisons. The problem is generally the opposite in most countries. In Italy, the ratio between effective inmates and the maximum admissible number according to EU rules was 113% in 2017 – see http://www.repubblica.it/solidarieta/diritti-umani/2017/07/31/news/carceri_in_italia_crescono_pericolosamente_sovraffollamento_e_suicidi-172043754/. The EU has recently fined Italy for prison overcrowding. Savings on EU fine costs are not added to the picture that therefore may underestimate actual benefits from the MIC project.

[8] Three years in prison after recidivism consider the average expected years in prison after recidivism of MiC beneficiaries and the other convicted people, duration, and drop-off rate of MiC. In our sensitivity analysis we check how our analysis changes when we consider 2 years of prison after recidivism. Note however that penalties for re-convicted are usually more severe.

initial investment (i.e., $\phi = 1.057$).

As always in impact studies, it is fundamental to evaluate whether project benefits are overestimated for not taking into account deadweight, crowding-out, attribution, and drop-off. In our analysis, the deadweight is represented by the complement of the recidivism rate without intervention. Therefore, it is already considered since we calculate project gain as the difference between the recidivism rate with and without the project. As well, there is no crowding out because we assume there are no other alternative projects to the standard public jail path in the absence of the MiC project. The result of the project can fully attributed to the treatment. The drop-off is already implicit in our assumption of average recidivism length (e.g., if the average length is three years the treatment has full effects for three years and 100 percent drop-off after them).

Table 1: Beseline scenario for a SIB in the Made in Carcere project

Item	**Value**
Yearly cost per inmate	€58,000
Bonus for penitentiary policemen	10%
Total operating costs per year	€200,000
Intertemporal discount rate	5%
Recidivism without SIB	70%
Recidivism in good state	5%
Recidivism in bad state	62%
Probability of good state	0.80
Probability of bad state	0.20
Average prison years of recidivists	3
Project length (years)	10
Yearly effect distribution (linear)	10%
Risk-free interest rate	0.434
Risk premium	0.32
Total costs (X)	€3,470,100.00
Outcome if success (Y)	€10,615,276.04
Outcome if failure (F)	€1,206,495.51.00
Profits share (π)	0.211
Guarantee fund (ϕ)	1.057
Government expected gain	€3,777,396.53
Private investors expected gain/Government risk factor	€1,506,023.40
Multiplier	€6.15

4.1.1 Sensitivity analysis

Our baseline scenario may be considered excessively optimistic when looking at critical parameters. We therefore perform a sensitivity analysis on the most relevant parameters: i) the outcome in case of failure; ii) fixed costs (raised from 200,000 to 400,000); iii) probability of success (reduced down from the 80 percent base assumption); iv) average years in jail post recidivism (reduced from 3 to 2 or 1 year only); iv) loss in bad state.

The first parameter we analyse is the outcome in case of failure. In our baseline scenario, we assume this failure outcome reflect a low reduction of recidivism (i.e., 62%), suggesting that if the project is not successful then the treated inmates re-offend almost as much as non-treated inmates. This is a pessimistic assumption, and therefore we relax it and assume that recidivism in case of failure is 50%. Accordingly, the guarantee fund becomes more attractive for the government as it is lower than 1, and the government expected gain also increases, without affecting any private investor expected outcome (Table 2, column 3).

The second parameter that we analyse it the fixed costs. It may be argued that these costs are low and that it would be reasonable to assume fixed costs equal to €400,000. When this occurs, we still obtain a feasible SIB with higher profit shares and higher guarantee fund. However, the increase in fixed costs is at the expenses of the government risk factor, which is now higher and requires more risk-seeking attitudes or higher difference between governmental and private costs.

We also find that the SIB scheme remains feasible when departing from the beseline scenario with the reduction of average recidivism years to 2 (Table 2, column 5). In this case the government needs to be less risk-seeking.

Table 2: Sensitivity analysis for a SIB in the Made in Carcere project

Item	**Baseline**	**Soft failure**	**Higher costs**	**Lower prison years of recidivism**
Yearly cost per inmate	€58,000	€58,000	€58,000	€58,000
Total operating costs per year	€200,000	€200,000	€400,000	€200,000
Bonus for penitentiary policemen	10%	10%	10%	10%
Intertemporal discount rate	5%	5%	5%	5%
Recidivism without SIB	70%	70%	70%	70%
Recidivism in good state	5%	5%	5%	5%
Recidivism in bad state	62%	50%	62%	
Probability of good state	0.80	0.80	0.80	0.80
Probability of bad state	0.20	0.20	0.20	0.20
Average prison years of recidivists	3	3	3	2
Project length (years)	10	10	10	10
Yearly effect distribution (linear)	10%	10%	10%	10%
Risk-free interest rate	0.434	0.434	0.434	0.434
Risk premium	0.32	0.32	0.32	0.32
Total costs (X)	€3,470,100.00	€3,470,100.00	€5,870,100.00	€2,913,400.00
Outcome if success (Y)	€10,615,276.04	€10,615,276.04	€10,615,276.04	€7,256,718.06
Outcome if failure (F)	€1,306,495.51	€3,266,238.78	€1,306,495.51	€893,134.53
Profits share (π)	0.211	0.211	0.537	0.291
Guarantee fund (ϕ)	1.057	0.493	1.211	1.127
Government expected gain	€3,777,396.53	€4,169,345.19	€335,796.53	€1,806,185.76
Private investors expected gain/Government risk factor	€1,506,023.40	€1,506,023.40	€2,547,623.40	€1,264,415.60
Multiplier	€6.15	€13.19	€1.24	€3.75

4.1.2 Further discussion

In our analysis, we strictly limit the project benefits to the foregone government cost of recidivism. However, the MiC project has other positive effects on the lives of the beneficiaries. First, women who do not re-offend may find a job and their wages proxy an additional project benefit. Similarly, they can contribute to the caring activities of their families therefore increasing their families well-being. Thus, a more comprehensive analysis should take these benefits into account. These are not benefits directly arising from the MiC project, but they are benefits that the government may be willing to pay for. Any time a public administration invests in active employment policies it is in fact implicitly 'spending' a given amount of money per job created (i.e., the ratio between total active employment policy expenditure and the number of jobs created). The additional indirect benefits may also explain the risk-seeking attitude of government.

4.2 The Health Budget project

The Health Budget (HB) project is a three-year personalised plan which operated in Campania, Italy, in the last decade. The project consists of a personalised care for patients with mental diseases who are offered alternatives to hospitalisation by a team of physicians and psychologists on patients with mental diseases. Based on a rich set of individual and collective information, the team designs personalised care plan for each patient and proposes innovative solutions such as work reintegration and social farming as an alternative to the standard care of hospitalisation. In particular, nowadays social farming offers widespread opportunities and also multinational corporations like Leroy Merlin took part to HB as a part of their CSR policy. The project has been initially tested on a target of 60 patients. In this first trial, the daily cost of three practitioners (psychologists and physicians) was €82 per patient, while the daily cost of the hospitalisation was €300 per patient. The project was fully successful, since none of the patients have been re-hospitalised in the following years.

In this section, we simulate a SIB which replicates the HB. Based on the first trial data discussed above (i.e., a daily cost of €82 per patients, 60 patients treated, and seven years project-length with 3 years of treatment and 4 years of follow-up), the total cost is €5,387,400. We prudentially assume the number of re-hospitalised patients equal to 50 percent of the targeted patients.[9] Therefore, the outcome of HB in case of success is €22,995,000, corresponding to the cost differential between SIB and non-SIB scenarios. We as well assume that the project is successful with probability 80 percent, while it fails with probability 20 percent. Note that gains are immediately available as the project takes patients away from the structure since its start. Similarly, consider that gains for the government should last for all the rest of the patient's life in case of full recovery. We however assume a very conservative hypothesis of lack of government benefits at the end of the three years. This implies either an implausible zero life expectancy after the end of the treatment or a more realistic hypothesis of partial rationing on the number of beds in hospital psychiatric structures (hence the benefit of bringing the treated patients away is reduced since another patient in the list enters the structure).

Under these assumptions, the model described in section 3 leads to a feasible solution of a SIB contract equal to $(\pi, \phi) = (0.133, 1.434)$, assuming a k-risk-seeking government or a difference between the government and the private investors costs higher that k, with $k = 2,338,131.60$ (Table 3).

4.2.1 Sensitivity analysis

In our sensitivity analysis we explore how the SIB changes as the assumptions on costs, outcome in case of failure, and share of successfully treated patients change. Table 4 shows that we have a feasible SIB under examined changes in our assumptions. In particular, if we assume the failure outcome being a fifth of the success outcome, then we have a guarantee fund less than 1 (Table 4, column 2). Moreover, if hospitalisation costs are higher (that is, potential gains are lower),

[9] This conservative assumption consider the difficulty of replicating the project on large scale that implies higher supply of external threats for the patients.

Table 3: Baseline scenario for a SIB in the Health Budget project

Item	**Value**
Daily cost per hospitalised patient	€300.00
Daily cost per treated patient	€82.00
Share of successfully treated patients	50%
Intertemporal discount rate	5%
Years of treatment	3
Years of follow-up	4
Risk-free interest rate	0.434
Risk premium	0.32
Total costs (X)	€5,387,400.00
Outcome if success (Y)	€22,995,000.00
Outcome if failure (F)	€0.00
Profits share (π)	0.133
Guarantee fund (ϕ)	1.434
Government expected gain	€10,670,468.40
Private investors expected gain/Government risk factor	€2,338,131.60
Multiplier	€7.91

Table 4: Sensitivity analysis for a SIB in the Health Budget project

Item	**Baseline**	**Soft failure**	**Higher costs**	**Higher successfully treated**
Daily cost per hospitalised patient	€300,000	€300,000	€200,000	€200,000
Daily cost per treated patient	€82,000	€82,000	€82,000	€82,000
Share of successfully treated patients	50%	50%	50%	70%
Intertemporal discount rate	5%	5%	5%	5%
Years of treatment	3	3	3	3
Years of follow-up	4	4	4	4
Risk-free interest rate	0.434	0.434	0.434	0.434
Risk premium	0.32	0.32	0.32	0.32
Total costs (X)	€5,387,400.00	€5,387,400.00	€5,387,400.00	€5,387,400.00
Outcome if success (Y)	€22,995,000.00	€22,995,000.00	€15,330,000.00	€21,462,000.00
Outcome if failure (F)	€0.00	€4,599,000.00	€3,066,000.00	€4,292,400.00
Profits share (π)	0.133	0.133	0.235	0.145
Guarantee fund (ϕ)	1.434	0.580	0.865	0.637
Government expected gain	€10,670,468.40	€11,590,268.40	€5,151,668.40	€10,302,548.40
Private investors expected gain/Government risk factor	€2,338,131.60	€2,338,131.60	€2,338,131.60	€2338131.60
Multiplier	€7.91	€19.54	€6.53	€16.00

then both the share of profits and the guarantee fund increase (Table 4, column 3), but this increase may be balanced by an increase in the share of successfully treated patients (Table 4,

Table 5: Potential conflicts of interest under the SIB scheme.

Variable	Government	Private investors	NGO	Solution
Project cost	Interest to reduce project costs to pay less guarantee fund	Interest to reduce project costs to increase project profits	Interest to inflate project costs (preventing unfeasible SIB) as they are proportional to their wages	Cost sharing for NGO
Project expected revenues	Interest to inflate revenues to let private investors be willing to accept less profits shares	Interest to reduce revenues to increase profits share	Interest to inflate revenues up to the trigger point that makes the SIB feasible	
Project expected risk	Interest to reduce risk to give lower profit share and lower guarantee fund coverage	Interest to increase risk in order to get higher profit shares or higher guarantee fund coverage	Interest to reduce risk up to the trigger point that makes the SIB feasible	Evaluation of the project risk from external evaluators
Choice of NGO	Interest for a politically friendly NGO regardless the efficiency, as votes are more important than public debt			NGO chosen by private investor

column 4).

5 Potential conflicts of interest under an asymmetric information scenario

The theoretical analysis on comparative statics and the simulated sensitivity analysis discloses several conflicts of interest that may occur if we relax the assumption of perfect information. Asymmetric information may arise under different respects, such as risk-return characteristics of the activity and quality and effort of the delegated organisation performing the social service.

As for the risk-return characteristics, the organisation performing the service could be interested in increasing project costs, as project costs are indeed revenues for the organisation (Table 5). An independent audit on project costs may be required by the government and private investors.

As for the quality and the effort of the organisation providing the service, government officials may be politically biased, that is, they may be interested in selecting the organisation ensuring the highest political benefits, which is not necessarily the best performer. This may happens because government officials do not directly incur in costs in case the project fails.[10] In this case, it is advisable that the selection of the organisation be in charge of private investors, who directly benefit from the success of the venture.

[10] In some legislations, civic servants can be prosecuted and found directly responsible with their own wealth for damages to public money. Even in that case the expected costs of their damage action may be low in case of poor efficiency of civil justice.

Similarly, private investors are interested to overstate project risks in order to negotiate a higher share of profits from the commissioner. On the contrary, the government and the organisation performing the service may have the interest to show that the project is feasible and private financing is profitable (Wong et al., 2016). As for the previous case, an audit of a third independent party of risks and returns of the project can overcome these problems.

Two additional conflicts of interest that can typically occur concern hidden actions of the service providers – when their effort cannot be monitored – and hidden information on project outputs. This does not apply on the direct output of the project, as in our case studies provided in section 4 with the example of jail recidivism where we know whether inmates re-offend, and the example of health budget where we know whether patients are re-hospitalised. Hidden information on project outputs may however apply on the benchmark output used to assess the success or the failure of the project. In other words, the benchmark output under the scenario without the SIB can be arguable. The problem of hidden actions of the provider may be overcome with some form of variable (i.e., performance based) payment to service providers; the problem of hidden information may be overcome with an ex ante agreement between commissioners and the intermediary on the counterfactual benchmark output (e.g., the regional average recidivism rate in the case of the jail recidivism).

6 Conclusions: what we have learnt

SIBs are innovative promising financial schemes involving several actors. Under the SIB scheme the most efficient and reputable organisation in the provision of a given social service that reduces government expenditure is hired by the government, and private investors participate to the venture by financing it with their funds. Investment risk is therefore rolled over them by the government that only partially covers the risk with a guarantee fund. In case of success, government gains in terms of reduced public expenditure are shared with private investors.

In this paper we show that the very (perfect information) SIB problem consists in the government and private investors contracting profit shares and the share of investment covered by a guarantee fund reimbursing private investor losses. The scheme is viable and convenient if both the government and private investors participation constraints hold. More specifically, the SIB scheme is convenient for private investors when their participation is equivalent to purchasing an equivalent asset not below the security market line; it is convenient for the government when the it ensures higher gains upon the alternative of direct financing. This can happen through the following mechanism: SIB leverages private capital transferring on it part of the risk, as it mobilises a limited share of government resources up to the amount of the guarantee fund. Our theoretical analysis under the perfect information benchmark shows that there is no SIB passing both government and private investor participation constraints if the government is risk averse or risk neutral and the realisation of the activity is not more expensive for the government than for the private investors. In a second scenario we relax this last assumption and show that, when bureaucratic costs or X-efficiency in the public sector make its costs higher than those of the private sector, a SIB is feasible even with a risk neutral or risk averse government.
In the second part of the paper, we apply the SIB structure by simulating the replication on larger scale of the figures of two projects realised in the past that have particularly promising features (i.e., a project aimed to prevent jail recidivism and a health budget project). We find the conditions under which, given current standard cost parameters for service provision by the government, SIB schemes for the two projects are viable in that they ensure risk-adjusted profits not below the security market line for private investors, while meeting at the same time the government participation constraint.

The SIB is a complex and articulated infrastructure involving actors with different objective functions. Therefore, it requires well-designed governance and rules when we depart from the perfect information framework. In particular, we argue as advisable that the private investor takes part to the selection of the organisation performing the social task, in order to avoid political bias when the selection is performed by the government. We also consider that an audit of independent third parties is essential to ascertain project revenues, costs, return, and risk in order to avoid distortion in their evaluation by one of the involved parties for their own interest. We as well discuss other two imperfect information problems arising in the scheme such as hidden information on the final project outcome and hidden action of the organisation performing the social service.

Results of our paper provide a theoretical and empirical framework to develop and apply SIBs schemes to different types of social services and can stimulate further contributions in this novel field of the literature.

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A Appendix

Proof of equation (1).

$$
\begin{aligned}
\sigma_a(r) &= E_a[r^2] - E[r]^2 = \\
&= p\pi^2(\frac{Y-X}{X})^2 + (1-p)(\frac{F-(1-\phi)X}{X})^2 \\
&\quad -p^2\pi^2(\frac{Y-X}{X})^2 - (1-p)^2(\frac{F-(1-\phi)X}{X})^2 - 2p(1-p)\frac{Y-X}{X}\frac{F-(1-\phi)X}{X} = \\
&= p(1-p)\pi^2(\frac{Y-X}{X})^2 + p(1-p)(\frac{F-(1-\phi)X}{X})^2 - 2p(1-p)\frac{Y-X}{X}\frac{F-(1-\phi)X}{X} = \\
&= p(1-p)\Big(\pi^2(\frac{Y-X}{X})^2 + (\frac{F-(1-\phi)X}{X})^2 - 2\frac{Y-X}{X}\frac{F-(1-\phi)X}{X}\Big) = \\
&= p(1-p)\Big(\pi\frac{Y-X}{X} - \frac{F-(1-\phi)X}{X}\Big)^2
\end{aligned}
$$

□

Proof of the solution maximisation problem (4).

$$
\begin{aligned}
&\max_{\pi,\phi} && p(1-\pi)(Y-X) - (1-p)\phi X \\
\text{s.t.}\quad &(\text{Gc}): && p\pi(Y-X) + (1-p)(F-X+X\phi) = k \\
&(\text{Pc}): && p\pi(Y-X) + (1-p)(F-X+X\phi) \geq X(a_0 + a_1\sigma^2(\pi,\phi))
\end{aligned}
$$

(Pc) must be binding since the government wants to set its risk seeking factor at the minimum. Thus, the problem writes

$$
\begin{aligned}
&\max_{\pi,\phi} && p(1-\pi)(Y-X) - (1-p)\phi X \\
\text{s.t.}\quad &(\text{C}): && p\pi(Y-X) + (1-p)(F-X+X\phi) = X(a_0 + a_1\sigma^2(\pi,\phi))
\end{aligned}
$$

Constraint (C) is a parabola as a function of ϕ, and the solutions are given by

$$
\pi^*_{\pm} = [0,1] \cap [-\frac{X(-2a_1\sqrt{p}(1-p)(1-\phi) \pm \sqrt{p - 4a_1(1-p)(1+a_0-\phi)} + \sqrt{p})}{2(a_1\sqrt{p}(1-p)(X-Y)}, 1].
$$

Plugging $\pi^*_{\pm}$ into the maximand function we obtain the optimal ϕ imposing the derivative equal to zero, that leads to

$$
\begin{aligned}
\phi^* &= \frac{X - F + a_0 X}{X} \\
\pi^*_+ &= \frac{X + a_1(1-p)a_0 X}{(Y-X)a_1(1-p)} \\
\pi^*_- &= \frac{a_0 X}{(Y-X)}
\end{aligned}
$$

Note that we are interested in solutions such that $\pi^* \in [0,1]$ and $\phi^* \geq 0$. Since $\pi^*_- \geq 0$ and $\pi^*_- \leq \pi^*_+$, in order to have a feasible solution we require $(1+a_0)X \leq Y$. If we also have $X(1 + a_1(1-p)(a_0+1)) \leq Y a_1(1-p)$, then we have two feasible solutions, that is π^*_- and π^*_+. Similarly, we require $\phi \geq 0$, that is $F \leq (1+a_0)X$. □

Proof of (8) *and Proposition 2.* The participation constraints are now

$$
\begin{aligned}
p(Y - X_G) + (1-p)(F - X_G) &\leq p(1-\pi)(Y - X_P) - (1-p)\phi X_P + k \\
p\pi(Y - X_P) + (1-p)(F - X_P + \phi X_P) &\geq X_P(a_0 + a_1\sigma^2(\pi,\phi))
\end{aligned}
$$

$$
\begin{aligned}
p(X_P - X_G) + p\pi(Y - X_P) + (1-p)(F - X_G + \phi X_P) &\leq k \\
p\pi(Y - X_P) + (1-p)(F - X_P + \phi X_P) &\geq X_P(a_0 + a_1\sigma^2(\pi,\phi))
\end{aligned}
$$

$$
\begin{aligned}
p\pi(Y - X_P) + (1-p)(F - X_G + X_P - X_P + \phi X_P) &\leq k - p(X_P - X_G) \\
p\pi(Y - X_P) + (1-p)(F - X_P + \phi X_P) &\geq X_P(a_0 + a_1\sigma^2(\pi,\phi))
\end{aligned}
$$

$$
\begin{aligned}
p\pi(Y - X_P) + (1-p)(F - X_P + \phi X_P) + (1-p)(X_P - X_G) &\leq k - p(X_P - X_G) \\
p\pi(Y - X_P) + (1-p)(F - X_P + \phi X_P) &\geq X_P(a_0 + a_1\sigma^2(\pi,\phi))
\end{aligned}
$$

$$
\begin{aligned}
p\pi(Y - X_P) + (1-p)(F - X_P + \phi X_P) &\leq k + (X_G - X_P) \\
p\pi(Y - X_P) + (1-p)(F - X_P + \phi X_P) &\geq X_P(a_0 + a_1\sigma^2(\pi,\phi))
\end{aligned}
$$

Then, the government risk factor now requires $k \geq X_P(a_0 + a_1\sigma^2(\pi,\phi)) - (X_G - X_P)$, which proves (8). Then, the risk factor k is negative if and only if $X_P(a_0 + a_1\sigma^2(\pi,\phi)) \leq (X_G - X_P)$, and this requires $X_G > X_P$ since $X_P(a_0 + a_1\sigma^2(\pi,\phi)) > 0$. This proves Proposition 2. □