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The Lemons Problem in a Labor Market with Intrinsic Motivation: When Higher Salaries Pay Worse Workers*

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Abstract

We study the Lemons Problem when workers have private information on both their skills and their intrinsic motivation for the job offered by firms in the labor market.

We first show that, when workers are motivated, inefficiencies due to adverse selection are mitigated. More interestingly, depending on the association between productivity and motivation, higher salaries affect the pool of candidates in three possible ways: they can attract (i) more skilled but less motivated applicants, as expected; (ii) more skilled and more motivated applicants; (iii) less skilled and less motivated applicants. The last two counterintuitive effects can only occur when a positive correlation exists between productivity and motivation.

Our results are relevant in the policy debate on whether it is possible to improve the quality of workers in vocational markets by changing their wage rate and reconcile the different empirical evidence provided so far on motivated workers such as public servants, teachers, health professionals and, politicians.

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1 Introduction

Higher wages may be necessary to attract applicants with higher skills, but does it come at the cost of attracting an applicant pool that is less motivated for the task to be performed? If intrinsic motivation is a significant determinant of vocational job performance, as a recent literature suggests (Handy and Katz 1998, Francois 2000, Heyes 2005, Delfgaauw and Dur 2007), providing higher salaries may not help to improve the efficiency and the quality of sectors where workers' intrinsic motivation matters.

The literature has discussed the problem of how to attract high quality applicants, both theoretically and empirically, focusing on specific vocational markets: many papers have analyzed the market for teachers (among others Figlio and Stone 1997, Figlio and Lucas 2000, Figlio 1997, Nickell and Quintini 2002, Dolton and Marcenaro-Gutierrez 2011), others the market for nurses (for example Heyes 2005 and Barigozzi and Turati 2012), more recently great attention has been paid to the market for civil servants (Francois 2000, 2007, Prendergast 2007, Besley and Ghatak 2005, Delfgaauw and Dur 2008, 2010 and Macchiavello 2008) and to the market for politicians (Besley 2004, 2005, Caselli and Morelli 2004, Messner and Polborn 2004, Keane and Merlo 2007, Mattozzi and Merlo 2008, Ferraz and Finam 2011).

Most of the empirical papers focus on workers' skills and the prevailing evidence suggests that higher wages increase applicants' ability; nevertheless important exceptions are Merlo *et al.* (2009) and, Kean and Merlo (2007), who find different results. The empirical question concerning motivation is obviously more difficult to address and to the best of our knowledge only Dal Bò *et al.* (2013) analyze how skills *and* intrinsic motivation characterizing the pool of applicants are affected by a wage increase. Interestingly, they show that higher salaries attract more skilled workers and *do not* decrease applicants' motivation. A more detailed description of the relevant empirical literature is provided in the separated section which follows.

Our paper theoretically investigates the (self)selection of applicants in a sector where workers' intrinsic motivation matters, referring to the well known Lemons Problem (Akerlof 1970). As will be explained in the following subsection, our results reconcile the empirical evidence obtained so far.

The Lemons Problem applied to the labor market (see for example Mas-Colell *et al.* 1995, chapter 13), studies market inefficiencies arising when firms in a competitive market offer a job to workers who have private information on their productivity level. As is well known, in the standard model productivity is the only characteristic of the worker and efficiency would require the firms to offer a different wage to workers characterized by different productivity levels. However, since productivity is not observable to the firms, the latter offer a uniform wage to all the workers. The workers' outside option, on the other hand,

is contingent on workers' ability. As is well known, this implies that workers' decisions about accepting the job or not depend on their productivity level in a way that adversely affects the firms: outside the market the payoff is larger for more productive workers, thus only relatively less capable workers are willing to accept the job at any given uniform wage offered by firms. This leads to an inefficiently low employment rate and to a low average productivity of active workers. What is crucial for our analysis is that, as we expect, efficiency improves when the wage rate increases since more productive workers enter the market. In particular, in the standard model without intrinsic motivation the average productivity of active workers is monotonically increasing in the wage rate.

Let's now consider a vocational market and suppose that workers are also characterized by their level of intrinsic motivation for the job offered by firms. How does this second source of workers' private information affect the Lemons Problem? How is labor supply characterized in this vocational market? In particular, do higher salaries pay more productive workers, as in the standard model? Does this occur at the cost of attracting candidates with lower intrinsic motivation?

We will show that the association properties of productivity and motivation levels in the population of potential workers dramatically affect the composition of the active work-force. In particular, three different outcomes are possible, which interestingly correspond to the different pieces of evidence documented so far by the empirical literature. As we will illustrate, our results provide new insights into the ongoing debate on how to increase the offer and the quality of workers belonging to vocation-based markets.

We interpret intrinsic motivation as a benefit from being employed in a vocation-based sector, unrelated to workers' effort or output (see also Heyes 2005 and Delfgaauw and Dur 2010). A "vocation-based labor market" is thus a sector where workers' intrinsic motivation can be relevant, as in the market for civil servants, nurses, teachers and, politicians; whereas a standard, non vocational labor market is a sector where workers' intrinsic motivation does not matter.

In our framework, as in the standard model of adverse selection in markets, (i) the opportunity cost of accepting the job (or the workers' outside-option) is increasing in workers' productivity; (ii) for informational or institutional reasons firms in the market offer a uniform wage (i.e. a salary that is independent of productivity and vocation).[1]

We first show that, given the wage rate, average productivity of active workers is higher in the vocation-based sector than in the non-vocational one. This occurs since, all else being equal, a vocation-based market can attract some high-productivity workers, characterized by high vocation. Thus, with respect to the standard case of a non-vocational labor market, intrinsic motivation mitigates the production inefficiency due to adverse selection.

More interesting are results concerning how average productivity and average vocation of active work-

[1] The assumption of a uniform wage is however relaxed in the Appendix 7.4.

ers change with respect to the wage rate. As already mentioned, in a standard market, if wages increase average productivity among workers hired by firms monotonically increases. In the same vein, in the vocational market and corresponding to a salary increase, we expect average productivity to rise (since more productive workers are interested in the job offered by firms), and average vocation to fall (since less motivated candidates apply for the job). In other words, our intuition is that higher wages are necessary to attract higher skills applicants but that this comes at the cost of attracting an applicant pool that is less motivated for the job.

Importantly, in the paper we show that, together with the expected results, one of two counter-intuitive effects can emerge: when the wage rate increases either average productivity of active workers can fall or average vocation can rise for at least an interval of possible wage levels. A necessary and sufficient condition allowing for counterintuitive effects is the existence of at least a level of salary such that active workers have average productivity above the average of the whole population, average motivation below the average of the whole population, or both of them.[2]

We show that such condition is consistent with a positive association between productivity and vocation in the population of potential workers and, thus, the two counterintuitive effects cannot be observed together. Moreover, the necessary and sufficient condition can be reinterpreted as a simple necessary condition when a linear dependence in mean exists between the two variables. In fact, provided we assume a positive correlation between skills and motivation, the counter-intuitive effect we observe depends on the magnitude of the slope of the outside-option function, compared to the (positive) slope of the regression line. In particular, if the slope of the outside-option function is lower than the slope of the regression line, average productivity of active workers can be decreasing in the wage rate for at last an interval of possible wage levels; whereas if the opposite holds, average vocation of active workers can be increasing in the wage rate.

To understand why, note that the outside-option function determines the shape (and thus the slope) of the curve of marginal workers, whereas the uniform salary offered by firms defines the position of the curve of marginal workers on the set of potential workers (and thus defines who accepts the job). Let's consider the case where the slope of the outside-option function is lower than the slope of the regression line. This situation implies that workers with high-skills and high-motivation have low reservation wage on average thus entering the market first. In particular, a positive marginal change in the salary may bring into the market workers with both skills and motivation below average.

It is worth noting that, because of the positive correlation between the two variables, the two coun-

[2] In a companion paper (Barigozzi and Turati 2012), one of us considers a simpler and discrete version of the present model with four worker types. Interestingly, in the four-types case one of the two counter-intuitive effects *always* occurs for a subset of possible wage rate levels; which one occurs depends on the relative impact that productivity and vocation have on the workers' reservation wages, which in turn affects the ranking of workers' reservation wages.

terintuitive effects cannot occur together and in particular, an increase in the wage makes both average productivity and vocation of active workers jointly increase or jointly decrease. In the first case, we observe that the pool of active workers improves with respect to both workers' characteristics, whereas, in the second case we observe that the pool fully deteriorates. Our findings thus suggest that, the lower the positive correlation between the two variables and the larger the slope of the outside option function (that represents the level of incentives characterizing the compensation scheme in the alternative sector), the more likely the improvement of the pool of active workers.

To summarize our results, depending on the association properties between productivity and motivation in the population of potential workers, three cases can be observed. (i) The intuitive effect, in which higher wages attract higher skilled but less motivated applicants: in this case average productivity of active workers is increasing whereas average vocation is decreasing in the wage rate. (ii) The counterintuitive case for the average productivity of active workers, in which higher salaries attract worse applicants: here both average productivity and average vocation of active workers are decreasing in the wage rate. (iii) The counterintuitive case for the average vocation of active workers, in which higher wages attract better applicants: both average productivity and average vocation of active workers are increasing in the wage.

Thus, our model is able to explain and reconcile the different empirical evidence that has been provided so far concerning the type of correlation existing between the wage rate and the quality of applicants in vocation based markets (see the subsection 1.1 below).

In the appendix we also present some evidence that a positive dependence conditional on the wage rate exists between productivity and motivation using data provided by the Italian survey ICSI 2007 (Indagine sulle Cooperative Sociali Italiane, that is, the Survey on Italian Social Cooperatives). The survey focuses on workers of Italian nonprofit firms. This empirical evidence, together with the empirical literature illustrated in subsection 1.1, shows that, in the real world, the counterintuitive phenomena described in our paper exist.[3]

In a second appendix, we show that our results can be easily extended to the case where, in the vocation-based market, salary is not uniform but depends on the workers' productivity (as the salary outside the vocation-based market does). To study the impact of a non-uniform salary we take the natural case in which incentives are higher outside the vocation based market than inside (according to the wage differential documented between private and public sectors and between for profit and not for profit ones); or the salary *outside* the vocation based market increases more in productivity than the salary *in* the vocation-based market.

[3]It is already well established that a positive association between workers' productivity and motivation exists in the case of civil servants. Petrovsky (2009), Naff and Crum (1999), Park and Rainey (2008), Ritz (2009) and Steijn (2008) find that public service motivation is positively correlated with job performance in the public sector.

1.1 Reconciling different empirical evidence

The more desirable outcome concerns the realization of the counterintuitive effect on average vocation of active workers, corresponding to the case in which higher salaries attract more skilled and more motivated applicants. This phenomenon is consistent with the empirical evidence provided by Dal Bò *et al.* (2013) who refer to the market for public servants. They study a recruitment drive for public sector positions in Mexico. Interestingly, in the experiment both applicants' intellectual ability and motivation are accurately documented. Intellectual ability is directly measured by the applicants' IQ index and indirectly by their current earning in the market (the applicant's outside opportunity). Motivation, on the other hand, is defined as the "inclination towards the public sector employment" and is measured using Perry's 1996 scale of Public Service Motivation.

The authors show that higher wages attract more skilled applicants, measured by their IQ index, and they find no evidence of adverse selection effects on motivation.[4] Results from this field experiment thus describe a case where higher wages attract workers that are better in terms of ability and are not worse in terms of motivation.

On the other hand, Merlo *et al.* (2009) provided evidence of the opposite phenomenon, by studying the labor market of Italian Politicians. Between 1985 and 2004, the average real total annual income of Italian legislators grew at an average annual growth rate of 3,8% making politics in Italy an highly lucrative job. Despite this sharp wage increase in the past twenty years, the quality of politicians has dramatically decreased (for example, today representatives are much less educated than in the past and have a lower outside opportunity). The authors thus document a negative correlation between the quality of elected legislators and the parliamentary wage. They also consider the degree of participation in parliamentary activity which we could interpret as a proxy for the motivation of the pool of applicants and they find no evidence of a positive effect of the wage increase on the degree of participation in parliamentary activity. Thus, their results seem in line with the most relevant outcome described in our paper, i.e. that higher salaries can attract worse workers.

Kean and Merlo (2007) analyzed how the career decisions of U.S. legislators respond to monetary incentives. They find that a reduction in the congressional wage would disproportionately induce "skilled" politicians (those who are more likely to win elections) to exit the Congress, but not politicians who are the "achiever" type (those who are characterized by ambition or desire for legislative accomplishment). Since the latter characteristic is a better measure of political quality, the authors conclude that the congressional wage does not impact on career decisions of high vs. low quality members of Congress, although it does affect skilled politicians relatively more.

[4] The authors find strong evidence that higher wages attract individuals showing higher levels of reciprocity, of engagement in prosocial behaviors and, higher willingness to cooperate.

As previously mentioned, Merlo *et al.* (2009) deals with Italian politicians entering Parliament. However, at the local level, evidence from Italy is different. Gagliarducci and Nannicini (2013) used data from Italian municipal governments from 1993 to 2001 showing that higher wages attract more educated and more efficient candidates. In the same line Ferraz and Finam (2011), examining Brazil's municipalities, showed that higher salaries attract candidates that are more educated and have more experience.

Since those papers do not explicitly measure the politicians' motivation, their evidence is consistent with both the intuitive situation in which average productivity of active workers increases whereas average motivation decreases, and with the counterintuitive instance where both average motivation and average productivity of active workers increase (as for Dal Bò *et al.* 2013).

As further evidence which is again consistent with the previous two scenarios, we report results from Dolton and Marcenaro-Gutierrez (2011). Studying the variation in teachers' pay across OECD countries and its significance for educational outcome, they showed a clear statistical association between higher relative teachers' salaries and higher standardized pupil scores across countries. They conclude that better wages attract teachers with higher degrees and improve pupil performance. As a consequence they suggest increasing teacher salaries as a policy measure to help schools to recruit and retain the higher ability teachers.

The rest of this paper is organized as follows. Section 2 describes the model. In Section 3 we study adverse selection in the vocation-based market. In Section 4 we analyze labor supply by investigating how average productivity and average motivation of active workers change with the wage rate. In Section 5 we provide some simulations reproducing intuitive and counterintuitive phenomena. We offer economic intuitions for the counterintuitive effects in Subsection 5.1 and discuss policy implications in Subsection 5.2. We also make the link between results on adverse selection and counterintuitive effects concerning labor supply in Subsection 5.3. Finally Section 6 concludes. In the Appendix 7.4 we extend our results to the case of a non uniform wage whereas in Appendix 7.3 we show evidence of a positive dependence between productivity and vocation in data provided by the Italian survey ICSI 2007.

2 A simple labor market model with workers' private information on skills and motivation

We consider a simple version of the model of adverse selection in the labor market, and we enrich the baseline set up by introducing workers' intrinsic motivation. The risk neutral (potential) workers are heterogeneous with respect to both their productivity and their intrinsic motivation for the job offered by firms in the vocational sector. We have in mind a labor market defined by two sectors: a

"vocational sector" (for instance, the market for health professionals, teachers or civil servants) where motivated workers obtain a vocational premium together with their salary and a "non-vocational sector", representing the workers' outside-option, where workers only receive a monetary compensation.

Since we focus on the supply side of the vocational market, we do not explicitly model firms' behavior.

Workers' productivity is interpreted as the number of units of output θ they produce if hired by firms in the vocational sector, with $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$, and $0 < \underline{\theta} < \bar{\theta}$.

Intrinsic motivation for the job offered in the vocational-sector is denoted by γ and it affects the participation constraint of the workers, as will be discussed later. We also assume that $\gamma \in \left[\underline{\gamma}, \bar{\gamma}\right]$ and $0 = \underline{\gamma} < \bar{\gamma}$.[5]

Let $F(\theta, \gamma)$, $H(\theta)$ and $G(\gamma)$ be respectively the cumulative joint distribution function and the marginal distribution functions of the population of potential workers, and let $f(\theta, \gamma)$, $h(\theta)$ and $g(\gamma)$ be their corresponding probability density functions. Average productivity and average vocation are thus $\mathrm{E}[\theta] = \int_{\underline{\theta}}^{\bar{\theta}} \theta h(\theta)\, d\theta = \mu_\theta$ and $\mathrm{E}[\gamma] = \int_{\underline{\gamma}}^{\bar{\gamma}} \gamma g(\gamma)\, d\gamma = \mu_\gamma$ respectively.

Workers aim at maximizing their earnings from their labor (in units of the numeraire good). A worker can choose to work either in the vocation-based labor market or outside: in particular a worker with productivity θ can obtain $r(\theta)$ in the non-vocational market. Thus, $r(\theta)$ is the opportunity cost to a worker with productivity θ of accepting employment in the vocation-based sector. The function $r(\theta)$ can be interpreted as the salary reached outside the vocation-based market. What is relevant for our purpose is that the outside option uniquely depends on the worker's productivity level.

The workers' outside option $r(\theta)$ is a strictly increasing function, i.e. more productive workers in the market are also more productive outside. We also assume that workers in the vocational sector receive a uniform wage w. This assumption has typically two justifications: first, productivity and vocation are workers' private information and no screening mechanisms are used by firms; secondly, in many vocational sectors contracts are mostly standardized and based upon a uniform wage policy (for instance, teachers in public schools or nurses in public hospitals), characterized by a series of pre-established steps on the career ladder. The assumption that firms offer a uniform salary in the vocation-based market will be relaxed in the Appendix 7.4.

Potential applicants accept the job if the total monetary benefit they receive from the job is larger than their outside option. By slightly abusing notation, the parameter γ also corresponds to the monetary

[5] Our setting can be considered a reduced form of Delfgaauw and Dur's set up (2010), where the two sectors are a private and a public one, respectively. In their model workers' characteristics are fully observable, the two sectors are perfectly competitive and, the outcome produced in the two markets depends on both workers' productivity and effort. By normalizing intrinsic utility for working in the private sector to zero, the authors interpret γ as the *relative* intrinsic preference for working in the public sector. Since they also assume that γ is sufficiently prevalent (or, that the demand for the outcome produced in the public sector is sufficiently low), the authors can restrict the domain of γ to be non-negative (see Delfgaauw and Dur 2010, page 656).

equivalent of the vocational premium workers obtain from their job in the vocation-based sector.

Several ways of modeling workers' intrinsic motivation can be found in the literature. Following Heyes (2005) and Delfgaauw and Dur (2010), we interpret intrinsic motivation as a benefit from being employed in a vocation-based sector, unrelated to workers' effort or output.[6]

In our model the total monetary benefit to the worker is given by the wage rate w plus the monetary equivalent of the vocational premium γ. As a consequence, a potential applicant with characteristics (θ, γ) accepts the job if and only if he/she receives a total benefit of at least $r(\theta)$ in the market (for convenience, we assume that the worker accepts in case he/she is indifferent):

$$r(\theta) \leq w + \gamma \tag{1}$$

Note that the vocational premium γ is uniquely obtained when the worker is hired by firms in the vocation-based market.

Inequality (1) shows that, all else being equal, the higher the worker's vocation, the higher the *total benefit* from the job in the vocational sector.[7]

From inequality (1) we observe the following two phenomena:

1. potential workers with high vocation are more likely to accept the job.

2. potential workers with low productivity are more likely to accept the job.

While the second phenomenon is the same as in the standard model of adverse selection and leads to the well known *adverse selection effect on productivity*, the first one is peculiar to the vocation-based labor market and has already been emphasized in Heyes (2005) and in Delfgaauw and Dur (2007). However, the two phenomena together, that is, the Lemons problem with bidimensional private information, has not been considered in the previous literature.

Importantly, in the standard model of adverse selection, inequality (1) simply reads $r(\theta) \leq w$. Note that the standard model can be interpreted as a labor market where intrinsic motivation is not active (or the parameter γ has no consequences on the worker's participation constraint). In practice, here vocation is *irrelevant* both in the market where the uniform salary w is offered and in the alternative one. Thus, in the standard model, the population of potential workers can be described by the same probability density function $f(\theta, \gamma)$ we introduced in our setting, provided that intrinsic motivation does not lead to any vocational premium when workers accept the job.

[6] Other papers describe motivated workers as people enjoying their personal contribution to the outcome produced in the vocational market (Besley and Ghatak 2005, Delfgaauw and Dur 2007, 2008, Barigozzi and Burani 2013).

[7] Note that, provided his/her vocation is sufficiently high and/or his/her outside option sufficiently low, a worker can decide to accept the job in the vocational sector even when the salary is $w = 0$. This corresponds to the case of volunteers workers or workers engaged in charity work.

Interestingly, a motivated worker may also deliver services of better quality. Consider for example a nurse asked to administer a particular number of injections; if she/he is motivated, she/he will probably give those injections with more "tender loving care". In the same way, a lecturer can be contracted to deliver a course, but availability for hallway conversation, motivating students and reinforcing the learning process, is essentially voluntary and typically attributed to teachers with a vocation (Heyes 2005, page 562). At school, a motivated teacher is generally more likely to promote curiosity and creative thinking and refine students' oral and written communication skills, even if this goes behind his/her explicit duties.

As these examples show, motivated workers may voluntarily provide a better quality of services which is typically non-contractible (although it is observable by recipients of services).[8] Hence, one can interpret intrinsic motivation as a valuable characteristics having a positive impact on workers' overall performance, independently of the number of units of output θ. An active worker characterized by both high productivity θ and motivation γ is thus a "better worker" who contributes to social welfare through larger output, better quality and higher workers' surplus.

3 The Lemons Problem in the vocation-based labor market

In this section we compare the inefficiency caused by adverse selection in the vocation-based and in the standard market.

Since the outside option $r(\theta)$ is increasing, the Lemons Problem both in the standard as well as in the vocational market arises. In fact, the workers' decision depends on their productivity level θ in a way that adversely affects the firms: the outside option is larger for more capable workers ($r'(\theta) > 0$), thus only relatively less productive workers are willing to accept the job at any given wage. A consequence is that average productivity of active workers is inefficiently low.

Note that $r'(\theta) > 0$ implies that productivity in the alternative sector is rewarded. In particular, $r'(\theta)$ is related to incentives: some salary scheme to screen workers is in place in the alternative market. (In Appendix 7.4 we consider the case in which a similar incentive scheme is used in the vocational market as well.)

From now on, let's call *marginal workers* those workers who are indifferent between accepting or not the job. Moreover, let $\hat{\theta}$ be the productivity of marginal workers in the standard model: $\hat{\theta}$ such that $r(\hat{\theta}) = w$. Thus $\hat{\theta} = r^{-1}(w)$. In the same way, let $(\tilde{\theta}, \tilde{\gamma})$ be the characteristics of marginal workers if vocation matters: $(\tilde{\theta}, \tilde{\gamma})$ such that $r(\tilde{\theta}) = w + \tilde{\gamma}$. Thus, $\tilde{\theta} = r^{-1}(w + \tilde{\gamma})$.

[8] Other examples of non-contractible quality levels in the market for health professionals are provided in McGuire (2008, page 281): "While time is one concrete candidate for what is meant by non-contractible quality as an input into patient health, diligence, responsiveness, and attentiveness can be thought of in the same category as well."

Definition 1 *Given a salary w_0, in the vocation-based market, the curve of marginal workers is:*

$$\gamma(\theta) = r(\theta) - w_0$$

In Figure 1, the set of potential workers is represented in the plane (θ, γ). The curve of indifferent workers splits the set of potential workers into two regions and, $r(\theta)$ and w_0 are such that $\theta_{\sup} = r^{-1}(w_0 + \bar{\gamma}) < \bar{\theta}$, where $\theta_{\sup}$ is the highest productivity level of workers accepting the job for a given salary w_0.[9] Obviously, the shadowed area indicates all types accepting the job given the salary w_0, the complementary region indicates types refusing the firms' offers.[10]

Insert Figure 1 here

Note that, in the standard model, given a salary w_0, active workers are simply indicated by the rectangle with sides $(r^{-1}(w_0) - \underline{\theta})$ and $\bar{\gamma}$ (see Figure 1).

Observation 1 *Given a salary w_0, both employment and average productivity of active workers are higher in the vocation-based market (VM) than in the standard market (SM): $E_{VM}[\theta\,|w_0] > E_{SM}[\theta\,|w_0]$.*

Proof. See Appendix 7.1. ■

As a main consequence, the lemon problem arises both in our setting and in the standard model. Furthermore, the previous lemma implies that adverse selection has a higher impact in the standard model in which motivation does not matter:

Corollary 1 *With respect to the standard model, workers' motivation reduces the inefficiency caused by adverse selection. In particular and conditional on a given wage rate w_0, the flatter the outside-option function $r(\theta)$, the smaller the level of inefficiency.*

Proof. Given all $\gamma \in [\underline{\gamma}, \bar{\gamma}]$, the larger $r^{-1}(w_0+\gamma)$, the larger the number of workers with $\theta > \hat{\theta} = r^{-1}(w_0)$ entering the market and, thus, the lower the production inefficiency due to adverse selection. Obviously, the amount of workers with $\theta > r^{-1}(w_0)$ entering the market is decreasing in $r'(\theta)$ (see again Figure 1). ■

Corollary 1 shows that the outside-option function $r(\theta)$ plays a crucial role in defining the extent of the difference between average productivity of active workers in a labor market with intrinsically motivated individuals and average productivity in the standard model. In particular, we observed that, if the slope of the curve is flat, then the average productivity of active workers in the vocation-based labor market is large.

[9] Depending on the function $r(\theta) - w_0$, $\theta_{\sup}$ can be lower or equal to $\bar{\theta}$. In other words, $\theta_{\sup} \equiv \min\{r^{-1}(w_0 + \bar{\gamma}), \bar{\theta}\}$.

[10] A similar graphical representation is provided in Delfgaauw and Dur (2010) in the case of full information on workers' characteristics and endogenous reservation wage (see also footnote 5).

The intuition behind this result can be stated as follows. Consider a set of potential workers with *high-ability* and *high motivation* and their decision whether to enter the vocation-based market or not at a wage rate w_0. Potential workers with high ability and high vocation receive the wage w_0 which does not reward their high level of productivity. Nevertheless they obtain their (high) vocational premium. When the reservation wage function is almost flat, the return of skills *outside* the vocation-based sector is low. This implies that, in the alternative sector, an increase in productivity has a minor impact on the workers' payoff. Thus, working in the vocation-based market becomes more attractive. When, on the contrary, the reservation wage function is steep, return to skills outside the vocation-based sector is high such that working in the vocation-based market becomes less attractive.

Note that, if the returns to skill outside the vocation-based labor market is completely flat ($r(\theta)$ is a horizontal line), production inefficiency due to adverse selection is the lowest possible. In particular, in this case, the salary is uniform both inside and outside the vocation based market.

Finally, consider the opposite extreme case in which returns to skills outside the market is infinitely high ($r(\theta)$ is a vertical line) and vocation has no impact on workers' participation constraint. In this case, vocation does not affect workers' decision on whether to accept the job. Thus, production inefficiency due to adverse selection is the same in the vocation-based as in the standard model.

4 How the wage rate affects the pool of active workers

We consider here how the characteristics of the active workers in the vocation-based market change as the wage rate increases.[11] This analysis will provide useful insights to understand how labor supply behaves in the vocation-based market.

Note that, as the wage rate marginally increases and shifts to $w_0 + dw_0$, the curve of indifferent workers $\gamma(\theta) = r(\theta) - w_0$ moves toward the bottom right side of Figure 1. Thus, as the wage rate increases, intuitively we expect firstly a negative impact on average vocation of active workers since also workers with vocation lower than before enter the market, and secondly a positive impact on average productivity, since also workers with productivity higher than before enter the market. However, the interplay of vocation and productivity in the workers' participation constraint determines workers' willingness to accept the job, so that also the counter-intuitive cases are possible: average vocation can be increasing and average productivity can be decreasing in the wage rate, as we will see.

We first show that average productivity of active workers is always increasing in the wage rate in the standard model. We will then analyze the case of the vocation-based sector.

[11] To justify that the workers' reservation wage $r(\theta)$ is fixed also when salary in the vocation-based labor market changes, we assume that the size of the vocation-based labor market is *small* with respect to the alternative sector. Thus, changes in price in the small sector do not affect price in the larger one.

Remark 1 *In the standard model, (i) the impact of a marginal increase in wage on marginal workers' productivity* $\hat{\theta} = r^{-1}(w_0)$ *is positive and equal to* $\frac{1}{r'(\hat{\theta})}$. *(ii) Average productivity of active workers is monotonically increasing in the wage.*

Proof. See the Appendix 7.2. ■

Remark 1 describes the positive impact due to a rise in the wage rate on average productivity of active workers in the standard model. When the salary increases, more productive workers accept the job and average productivity among active workers monotonically increases (see the dotted line in Figure 2). In particular, when $w = r(\underline{\theta})$, only subjects with characteristic $\underline{\theta}$ enter the standard non-vocational market and $\mathrm{E}_{SM}[\theta \,|w = r(\underline{\theta})] = \underline{\theta}$; whereas when $w = r(\bar{\theta})$ all workers enter the market so that $\mathrm{E}_{SM}\left[\theta \,|w \geq r(\bar{\theta})\right] = \mu_\theta$. Remark 1 states that, for wages included between $r(\underline{\theta})$ and $r(\bar{\theta})$, $\mathrm{E}_{SM}[\theta \,|w]$ is monotonically increasing in w (see also Mas-Colell *et al.* 1995, chapter 13).

Insert figure 2 about here

We now turn to the case in which potential workers are characterized by the two variables θ and γ. We will show that the association between the two variables is crucial in determining whether counterintuitive effects occur. We now define:

Definition 2 *Net reservation wage* $W(\theta, \gamma)$ *is the wage that makes the potential worker with characteristics* (θ, γ) *indifferent between accepting or not the job in the vocation-based market:* $W(\theta, \gamma) \equiv r(\theta) - \gamma$.

Since $r'(\theta) > 0$, the ranking of net reservation wages for types belonging to the vertex of the set of potential workers is the following: $W(\underline{\theta}, \bar{\gamma}) \equiv W_{\min} < \min\left\{W(\underline{\theta}, \underline{\gamma}), W(\bar{\theta}, \overline{\gamma})\right\} < \max\left\{W(\underline{\theta}, \underline{\gamma}), W(\bar{\theta}, \overline{\gamma})\right\}$ $< W(\bar{\theta}, \underline{\gamma}) \equiv W_{\max}$. In other words, workers with characteristics $(\underline{\theta}, \overline{\gamma})$ accept the job for the lowest wage rate and we observe that $\mathrm{E}_{VM}[\theta \,|w = W_{\min}] = \underline{\theta}$. Whereas workers with characteristics $(\overline{\theta}, \underline{\gamma})$ accept the job for the highest wage rate and $\mathrm{E}_{VM}[\theta \,|w \geq W_{\max}] = \mu_\theta$ since for $w \geq W_{\max}$ all workers enter the market. Of course, we cannot know a priori whether workers of type $(\underline{\theta}, \underline{\gamma})$ or of type $(\bar{\theta}, \overline{\gamma})$ enter the market first.

Note that reservation wages in the standard model are always higher than net reservation wages $W(\theta, \gamma)$, thus, the two wage rates that allow the first worker to enter the market and such that full employment is reached, are lower in the vocation-based market.

We will show that $\mathrm{E}_{VM}[\theta \,|w]$ can be decreasing in w for a subinterval in $]W_{\min}, W_{\max}]$. However, Observation 1 implies that, for every value of w, average productivity of active workers is slightly greater in the market where intrinsic motivation matters: $\mathrm{E}_{VM}[\theta \,|w] \geq \mathrm{E}_{SM}[\theta \,|w] \ \forall w \in [W_{\min}, W_{\max}]$. Thus, as Figure 2 shows, the function $\mathrm{E}_{VM}[\theta \,|w]$ necessarily lies above the function $\mathrm{E}_{SM}[\theta \,|w]$. The dashed curve shows the case in which $\mathrm{E}_{VM}[\theta \,|w]$ is monotonically increasing in the wage rate and the continuum

curve the case in which the counterintuitive effect for average productivity occurs for every value of the possible wage rates (or $\mathrm{E}_{VM}[\theta|w]$ is monotonically decreasing in the wage rate).

To understand why counterintuitive effects may occur, consider the following special case:

Example 1 ***Perfect correlation between*** θ ***and*** γ**.** *Suppose the correlation between the two variables is perfect, that is, all potential applicants can be represented through a line. Let us call* $(\theta_{\min}, \gamma_{\max})$ *the type belonging to the upper extreme point and* $(\theta_{\max}, \gamma_{\min})$ *the type belonging to the lower extreme point of the line when the correlation is* -1*. Using our definition of net reservation wage, it is* $W(\theta_{\min}, \gamma_{\max}) = r(\theta_{\min}) - \gamma_{\max}$ *and* $W(\theta_{\max}, \gamma_{\min}) = r(\theta_{\max}) - \gamma_{\min}$*, in which* $W(\theta_{\max}, \gamma_{\min}) > W(\theta_{\min}, \gamma_{\max})$*. In the same way let's call* $(\theta_{\min}, \gamma_{\min})$ *the type belonging to the lower extreme point and* $(\theta_{\max}, \gamma_{\max})$ *the type belonging to the upper extreme point of the line when the correlation is* $+1$*. Now* $W(\theta_{\min}, \gamma_{\min}) = r(\theta_{\min}) - \gamma_{\min}$ *and* $W(\theta_{\max}, \gamma_{\max}) = r(\theta_{\max}) - \gamma_{\max}$*, where* $W(\theta_{\min}, \gamma_{\min})$ *can be either higher or lower than* $W(\theta_{\min}, \gamma_{\max})$*.*[12] *Three possible cases can be observed:*

(i) The correlation between θ *and* γ *is* -1*. Suppose we progressively increase the wage rate* w_0 *in the interval* $[W(\theta_{\min}, \gamma_{\max}), W(\theta_{\max}, \gamma_{\min})]$ *such that the curve of marginal workers* $\gamma(\theta) = r(\theta) - w_0$ *shifts toward the bottom right side. Since net reservation wages are all aligned for types belonging to the line (and are increasing from* $W(\theta_{\min}, \gamma_{\max})$ *to* $W(\theta_{\max}, \gamma_{\min})$*), it is clear that by increasing wages, it is possible to attract workers with higher productivity and decreasing motivation. Thus,* $E_{VM}[\theta|w]$ *monotonically increases whereas* $E_{VM}[\gamma|w]$ *monotonically decreases in the wage rate. In other words, the intuitive effect occurs: higher wages attract workers with higher skills and lower motivation.*

(ii) The correlation between θ *and* γ *is* 1 *and net reservation wages of extreme types are such that* $W(\theta_{\min}, \gamma_{\min}) < W(\theta_{\max}, \gamma_{\max})$*, that is, worst workers have a lower net reservation wage than the best ones, meaning that* $r'(\theta)$ *is larger than the slope of the line where potential workers are located:* $\frac{\gamma_{\max} - \gamma_{\min}}{\theta_{\max} - \theta_{\min}}$*.*[13] *Again, net reservation wages are all aligned for types belonging to the line. As before, consider larger wage rates in the interval* $[W(\theta_{\min}, \gamma_{\min}), W(\theta_{\max}, \gamma_{\max})]$ *such that the curve of marginal workers shifts south-east. In this case, both* $E_{VM}[\theta|w]$ *and* $E_{VM}[\gamma|w]$ *monotonically increase with respect to the wage rate. Since, higher wages attract workers with both higher skills and intrinsic motivation, the counterintuitive effect occurs for intrinsic motivation.*

[12] In the Example we assume that the outside-option function $r(\theta)$ is such that it crosses the line where workers lie only once.

[13] In fact, let's consider two types (θ_1, γ_1) and (θ_2, γ_2) belonging to the line and such that $\theta_1 < \theta_2$ and $\gamma_1 < \gamma_2$. Since the two types belong to the line it necessarely follows that $\gamma_1 = \frac{\gamma_2 - \gamma_1}{\theta_2 - \theta_1}\theta_1 = \frac{\gamma_{\max} - \gamma_{\min}}{\theta_{\max} - \theta_{\min}}\theta_1$ and $\gamma_2 = \frac{\gamma_{\max} - \gamma_{\min}}{\theta_{\max} - \theta_{\min}}\theta_2$. If net reservation wages are such that $W(\theta_1, \gamma_1) < W(\theta_2, \gamma_2) \iff r(\theta_1) - \gamma_1 < r(\theta_2) - \gamma_2$, then it necessarely follows that $r(\theta_1) - r(\theta_2) > \frac{\gamma_{\max} - \gamma_{\min}}{\theta_{\max} - \theta_{\min}}(\theta_2 - \theta_1)$ or $\frac{r(\theta_2) - r(\theta_1)}{\theta_2 - \theta_1} \longrightarrow r'(\theta) > \frac{\gamma_{\max} - \gamma_{\min}}{\theta_{\max} - \theta_{\min}}$.

(iii) Finally consider the case in which correlation is 1 *and net reservation wages are such that* $W(\theta_{\min}, \gamma_{\min}) > W(\theta_{\max}, \gamma_{\max})$, *so that, the best workers have a lower net reservation wage and enter the vocational market first. This happens when* $r'(\theta) < \frac{\gamma_{\max}-\gamma_{\min}}{\theta_{\max}-\theta_{\min}}$. *By progressively increasing the wage rate in the interval* $[W(\theta_{\max}, \gamma_{\max}), W(\theta_{\min}, \gamma_{\min})]$ *we observe that both* $E_{VM}[\theta|w]$ *and* $E_{VM}[\gamma|w]$ *monotonically decrease in the wage rate. Since, higher wages attract workers with both lower skills and lower motivation, the counterintuitive effect occurs for applicants' skills.*

The previous example shows that the sign of the correlation between workers' motivation and skills is crucial in explaining counterintuitive effects. In particular, counterintuitive effects can be observed only when the correlation is positive. Moreover, the two counterintuitive effects do not occur jointly: in particular, either applicants' average skills are decreasing or average intrinsic motivation is increasing in the salary.

Finally, it is worth noting that the counterintuitive effect displayed by the model depends on the relative size of the slope of the curve of marginal workers $r'(\theta)$ and the slope of the line of potential workers $\frac{\gamma_{\max}-\gamma_{\min}}{\theta_{\max}-\theta_{\min}}$. The reason being that, if $r'(\theta) > \frac{\gamma_{\max}-\gamma_{\min}}{\theta_{\max}-\theta_{\min}}$, the worst worker is the first to enter the vocational market and then, as the wage rate increases, better types progressively enter as well. When the opposite inequality holds, the best worker is the first to enter the market and then worse workers progressively follow. We will show that those observations still hold in the case of a general joint probability distribution function.

To make our analysis more detailed, we now provide necessary and sufficient conditions that allow for counterintuitive effects in the general model, in which we consider the joint probability distribution for θ and γ. We can state the following remark:

Remark 2

- *A necessary and sufficient condition for* $E_{VM}[\theta|w]$ *decreasing in some interval of* $]W_{\min}, W_{\max}[$ *is that* $w_0 \in]W_{\min}, W_{\max}[$ *exists such that* $E_{VM}[\theta|w_0] > \mu_\theta$, *in fact* $E_{VM}[\theta|w]$ *converges to* μ_θ *if* $w = W_{\max}$.
- *A necessary and sufficient condition for* $E_{VM}[\gamma|w]$ *increasing in some interval of* $]W_{\min}, W_{\max}[$ *is that* $w_{00} \in]W_{\min}, W_{\max}[$ *exists such that* $E_{VM}[\gamma|w_{00}] < \mu_\gamma$, *in fact* $E_{VM}[\gamma|w]$ *converges to* μ_γ *if* $w = W_{\max}$.

Necessary and sufficient conditions that allow for counterintuitive results are stated in Proposition 1 below. We first introduce the following definitions:

Definition 3 $A_{1,w_0} \cup A_{2,w_0}$ *is the set of all workers entering the market at the salary* w_0. A_{1,w_0} *is the subset of workers with productivity below the average, i.e., such that* $\theta \leq E[\theta] = \mu_\theta$; *whereas* A_{2,w_0} *is the subset of workers with productivity above the average, i.e.,* $\theta \geq E[\theta] = \mu_\theta$. *As a main consequence,*

$E[\theta|A_{2,w_0}] > \mu_\theta$ *and* $E[\theta|A_{1,w_0}] < \mu_\theta$*. Furthermore, concerning the probability associated with each subset we observe that:* $P(A_{1,w_0}) = 1 - P(A_{2,w_0})$.

In the same way:

Definition 4 $B_{1,w_0} \cup B_{2,w_0}$ *is the set of all workers entering the market at the salary* w_{00}. $B_{1,w_{00}}$ *is the subset of workers with motivation below the average, i.e., such that* $\gamma \leq E[\gamma] = \mu_\gamma$*; whereas* $B_{2,w_{00}}$ *is the subset of workers with motivation above the average, i.e.,* $\gamma \geq E[\gamma] = \mu_\gamma$*. Thus,* $E[\gamma|B_{2,w_{00}}] > \mu_\gamma$ *and* $E[\theta|B_{1,w_{00}}] < \mu_\gamma$*. Moreover,* $P(B_{2,w_{00}}) = 1 - P(B_{1,w_{00}})$.

Proposition 1 ***(Necessary and sufficient conditions for counterintuitive results)***

- *If a wage* w_0 *exists such that*

$$\frac{P(A_{2,w_0})}{1 - P(A_{2,w_0})} \geq -\frac{E[\theta|A_{1,w_0}] - \mu_\theta}{E[\theta|A_{2,w_0}] - \mu_\theta}, \tag{2}$$

  then $E[\theta|w_0] > \mu_\theta$*, and at least a subset of* $[w_0, W_{\max}[$ *exists in which* $E[\theta|w]$ *is decreasing.*

- *If a wage* w_{00} *exists such that*

$$\frac{P(B_{1,w_{00}})}{1 - P(B_{1,w_{00}})} \geq -\frac{E[\gamma|B_{2,w_{00}}] - \mu_\gamma}{E[\gamma|B_{1,w_{00}}] - \mu_\gamma}, \tag{3}$$

  then $E[\gamma|w_{00}] < \mu_\gamma$*, and at least a subset of* $[w_{00}, W_{\max}[$ *exist in which* $E[\gamma|w]$ *is increasing.*

Proof. Let's consider the proof for the first part of the Proposition. The proof of the second part is equivalent and therefore omitted.

Consider the expected productivity of subjects entering the market for $w = w_0$,[14] $\mathrm{E}[\theta|w_0]$, that is,

$$\mathrm{E}[\theta|w_0] = \int_{\underline{\theta}}^{r^{-1}(\underline{\gamma}+w_0)} \int_{\underline{\gamma}}^{\bar{\gamma}} \theta p(\theta, \gamma|w_0)\ d\gamma\ d\theta + \int_{r^{-1}(\underline{\gamma}+w_0)}^{r^{-1}(\gamma+w_0)} \int_{r(\theta)-w_0}^{\bar{\gamma}} \theta p(\theta, \gamma|w_0)\ d\gamma\ d\theta$$

in which

$$p(\theta, \gamma|w_0) = \frac{p(\theta, \gamma)}{\int_{\underline{\theta}}^{r^{-1}(\underline{\gamma}+w_0)} \int_{\underline{\gamma}}^{\bar{\gamma}} p(\theta, \gamma|w_0)\ d\gamma\ d\theta + \int_{r^{-1}(\underline{\gamma}+w_0)}^{r^{-1}(\gamma+w_0)} \int_{r(\theta)-w_0}^{\bar{\gamma}} p(\theta, \gamma|w_0)\ d\gamma\ d\theta}$$

The law of iterated expectations allows us to write

$$\mathrm{E}[\theta|w_0] = \mathrm{P}(A_{1,w_0})\mathrm{E}[\theta|A_{1,w_0}] + \mathrm{P}(A_{2,w_0})\mathrm{E}[\theta|A_{2,w_0}]$$

where the two non overlapping subsets A_{1,w_0} and A_{2,w_0} described in Definition 3 are such that $A_{1,w_0} \cup A_{2,w_0}$ is the set of workers entering the market at a given salary w_0.

[14] Without loss of generality, here we consider the case defined both in Figure 1 and in Appendix 7.1.

Note that, given w_0, if the following condition holds,

$$\mathrm{E}[\theta|w_0] = (1 - \mathrm{P}(A_{2,w_0}))\,\mathrm{E}[\theta|A_{1,w_0}] + \mathrm{P}(A_{2,w_0})\mathrm{E}[\theta|A_{2,w_0}] \geq \mu_\theta \tag{4}$$

then the conditional expectation must be decreasing in the wage rate at least in a subset of $[w_0, W_{\max}[$.

Define $\epsilon_1 = \mathrm{E}[\theta|A_{1,w_0}] - \mu_\theta$ and $\epsilon_2 = \mathrm{E}[\theta|A_{2,w_0}] - \mu_\theta$ which are negative and positive numbers respectively. It is obvious that if (4) holds, then necessarely

$$(1 - \mathrm{P}(A_{2,w_0}))\,\epsilon_1 + \mathrm{P}(A_{2,w_0})\epsilon_2 > 0$$

which is equivalent to inequality (2). ■

Note that the left hand side of inequality (2) is always positive and unbounded, whereas the quantity on the right is positive and finite as long as $\mu_\theta < \infty$. So, a salary w_0 that satisfies the conditions may exist.

In the same way, the left hand side of inequality (3) is always positive and unbounded, whereas the quantity on the right is positive and finite as long as $\mu_\gamma < \infty$. So, again, an w_{00} that satisfies the conditions may exist.

The intuition of (2) is that, for a given wage rate w_0, we observe the counterintuitive effect on the average productivity of active workers if (i) the probability of observing highly productive workers $\mathrm{P}(A_{2,w_0})$ is greater than the probability of observing below-average productivity workers, $1-\mathrm{P}(A_{2,w_0})$; (ii) the ratio of the two previous probabilities is greater than the ratio of the distances of the two conditional averages, $\mathrm{E}[\theta|A_{1,w_0}]$ and $\mathrm{E}[\theta|A_{2,w_0}]$, with respect to the marginal mean. Thus, not only the probability $\mathrm{P}(A_{2,w_0})$ should be large enough to obtain a decreasing $\mathrm{E}[\theta|w]$, but we also need the average θ in A_{2,w_0} to be greater than μ_θ, while the average θ in A_{1,w_0} has to be close enough to μ_θ. In a nutshell, condition (2) requires a large number of applicants characterized by an average productivity sufficiently greater than the average μ_θ.

The intuition provided by (3) follows the same logic. In a nutshell again, it requires a sufficiently large number of applicants characterized by an average motivation sufficiently higher than the average μ_γ.

Note that both conditions are consistent with the hypothesis of strong positive association between θ and γ. In fact, condition (2) requires a large mass in the upper-right part of the set of potential workers, while condition (3) holds if a large mass is associated with the bottom-left part of the set of potential workers.

We now consider the two previous conditions (2) and (3) for the same wage level w_0, assuming that w_0 is sufficiently low for workers with average productivity and average motivation $\left(\theta = \mu_\theta, \gamma = \mu_\gamma\right)$ to remain out of the market (see also Figure 3).

Insert Figure 3 around here

The covariance between θ and γ conditional on the wage rate w_0 can be written as follows:

$$\begin{aligned} cov(\theta, \gamma \,|w_0\,) \;\; = \;\; & \mathrm{E}_{w_0}\left[(\theta - \mathrm{E}[\theta|w_0])\,(\gamma - \mathrm{E}[\gamma|w_0])\right] = \\ & \mathrm{E}_{w_0}\left[(\theta - \mathrm{E}[\theta|A_{2,w_0}])\,(\gamma - \mathrm{E}[\gamma|A_{2,w_0}])\right] P(A_{2,w_0}) + \\ & \mathrm{E}_{w_0}\left[(\theta - \mathrm{E}[\theta|B_{1,w_0}])\,(\gamma - \mathrm{E}[\gamma|B_{1,w_0}])\right] P(B_{1,w_0}) + \\ & \mathrm{E}_{w_0}\left[(\theta - \mathrm{E}[\theta|C_{w_0}])\,(\gamma - \mathrm{E}[\gamma|C_{w_0}])\right] P(C_{w_0}) \end{aligned} \tag{5}$$

where $C_{w_0} = A_{1,w_0} - B_{1,w_0} = B_{2,w_0} - A_{2,w_0}$. Given the definition of A_{2,w_0} and B_{1,w_0}, the first and the second terms of (5) are positive whereas the last term is negative. Conditions (2) and (3) imply that $P(A_{2,w_0})$ and $P(B_{1,w_0})$ are large, whereas $P(C_{w_0})$ is small. Thus, the two conditions together suggest that the covariance between θ and γ, conditional on the wage rate w_0 is likely to be positive. In other words and in line with the Example 1:

Remark 3 *Conditions (2) and (3) are consistent with a positive covariance between θ and γ conditional on the wage rate w_0.*

However, if the distribution of θ and γ in the population of potential workers is characterized by a positive covariance, then we must conclude that the two counter-intuitive effects cannot occur together:

Remark 4 *(i) Suppose that $cov(\theta, \gamma \,|w_0\,) > 0$. As the wage rate increases, $E[\theta|w > w_0]$ and $E[\gamma|w > w_0]$ must move in the same direction: either both increase or both decrease. (ii) Suppose that $cov(\theta, \gamma \,|w_0\,) \leq 0$. As the wage rate increases, $E[\theta|w > w_0]$ and $E[\gamma|w > w_0]$ must move in opposite directions: one increases whereas the other decreases.*

In fact, if condition (2) holds, then $\mathrm{E}[\theta|w > w_0]$ decreases such that, on average, workers with *productivity levels below average* are entering the market. For the same wage levels, $cov(\theta, \gamma \,|w_0\,) > 0$ implies that workers with *vocation below average* are entering the market. As a consequence $\mathrm{E}[\gamma|w > w_0]$ must decrease as well. In the same way, if condition (3) holds, then $\mathrm{E}[\gamma|w > w_0]$ increases such that, on average, workers with *motivation above average* are entering the market. For the same wage levels, positive conditional covariance implies that workers with *productivity above average* are entering the market. As a consequence $\mathrm{E}[\theta|w > w_0]$ must increase as well.

On the contrary, when the conditional covariance is negative, either $\mathrm{E}[\theta|w > w_0]$ is increasing and $\mathrm{E}[\gamma|w > w_0]$ is decreasing, or the opposite. In line with our intuition and through Example 1, we expect, when $cov(\theta, \gamma \,|w_0\,) \leq 0$, both intuitive effects to occur: $\mathrm{E}[\theta|w > w_0]$ is increasing and $\mathrm{E}[\gamma|w > w_0]$ is decreasing in the wage rate.

The next observation summarizes the results provided in this section.

Observation 2 *Suppose that* $cov(\theta, \gamma\,|w_0) > 0$, *then: if condition (2) holds,* $E[\theta|w]$ *and* $E[\gamma|w]$ *are both decreasing in the wage rate in a subinterval of* $[w_0, W_{\max}[$. *If, on the other hand, condition (3) holds,* $E[\theta|w]$ *and* $E[\gamma|w]$ *are both increasing in the wage rate in a subinterval of* $[w_0, W_{\max}[$.

To provide some empirical evidence on our findings, we also provide some results based on simulations.

5 Simulations

To show evidence of intuitive and counterintuitive phenomena, we set a Monte Carlo experiment. In particular, we provide some examples in which counterintuitive effects are possible both for expected productivity $\mathrm{E}[\theta|w]$ as well as for expected intrinsic motivation $\mathrm{E}[\gamma|w]$. Without loss of generality, we set $\underline{\theta} = 0$, $\overline{\theta} = 10$, $\underline{\gamma} = 0$ and $\overline{\gamma} = 5$. We also assume that the marginal expected values are $\mathrm{E}[\theta] = \frac{\underline{\theta}+\overline{\theta}}{2}$ and $\mathrm{E}[\gamma] = \frac{\underline{\gamma}+\overline{\gamma}}{2}$. Marginal standard errors have been chosen to keep the truncation rate of our Monte Carlo experiment lower than the 5%. We thus simulated different scenarios, by considering different slopes of the curve of marginal workers and different levels of correlation. To keep the intuition of our experiment as simple as possible, we consider a linear curve of marginal workers, that is, $r(\theta) = \kappa\theta$ and we also refer to the linear dependence between θ and γ described by $\gamma = a + b\theta + \tilde{\varepsilon}$, where $\tilde{\varepsilon}$ is an error term with zero mean and $b = \frac{cov(\theta,\gamma)}{var(\theta)}$. Obviously b is constant and its sign is determined by the sign of $cov(\theta, \gamma)$.

For each experiment we simulated a sample of 200,000 subjects with characteristic (θ_i, γ_i) from a bivariate Gaussian random variable defined on the joint domain of the two dimensions considered. Expected values of γ and θ given the wage level w are computed through Monte Carlo integration.

We first consider the case in which we observe an average productivity of active workers decreasing with respect to the wage rate, i.e., the situation in which we find the counterintuitive effect for average productivity. In this case we set $b = 1.7$ and $\kappa = 1.5$ thus assuming a lower slope of the line of marginal workers with respect to the regression slope. Panel A in Figure 4 shows that $\mathrm{E}[\theta|w]$ is decreasing as well as $\mathrm{E}[\gamma|w]$.

Insert Figure 4 about here

In the second scenario we keep the slope of the line of marginal workers $\kappa = 1.5$ unchanged, while we decrease the slope of the regression line such that $b = 0.6$. Thus, in this case, the line of marginal workers is steeper than the linear relation. Panel B in Figure 4 shows an increasing expected productivity and an increasing expected vocation. Thus, in this case we observe a counterintuitive behavior for the expected γ.

According to our idea, if we take the very same experiment and uniquely switch the sign of the covariance (from positive to negative), both intuitive effects occur. In particular we assume $b = -0.6$ and

$\kappa = 1.5$. In Panel C of Figure 4 we find support for an increasing average productivity and a decreasing average vocation of active workers.

Finally, as a further confirmation that counterintuitive results arise when b is positive and the relative magnitude of b and κ determines which type of counterintuitive result occurs, we consider the same slope for the regression line as in case B, $b = 0.6$, together with a lower coefficient, $\kappa = 0.4$. Panel D in Figure 4 show a decreasing average productivity and a decreasing vocational level as shown in case A above. Again, this result is counterintuitive with respect to the θ dimension.

Interestingly, all the simulations provide monotonic functions for both $\mathrm{E}[\theta|w]$ and $\mathrm{E}[\gamma|w]$. This means that, in the exercises, $cov(\theta, \gamma\,|w_0\,) > 0\ \forall w_0$ and either condition (2) or condition (3) are verified for all the possible values of the wage rate.

5.1 Economic intuitions

We now provide economic intuitions for the counterintuitive phenomena described in the simulations.

Let's begin with Case A, which is characterized by $\mathrm{E}[\theta|w]$ and $\mathrm{E}[\gamma|w]$ decreasing in the wage rate (see Figure 4, Case A). In Figure 5 the whole set of potential workers is represented, together with the average values of θ and γ, the regression line and the line of marginal workers defined by a wage rate set to 3. The line of marginal workers divides the set of active workers into two subsets, $A_{1,w_0=3}$ and $A_{2,w_0=3}$. As shown in Figure 5, condition (2) should be verified for $w_0 = 3$, $\mathrm{P}(A_{2,w_0=3})$ being clearly larger than $(1-\mathrm{P}(A_{2,w_0=3}))$. As already mentioned, the monotonicity of $\mathrm{E}[\theta|w]$ and $\mathrm{E}[\gamma|w]$ in Figure 4 (Case A) implies that condition (2) is verified for all other values of the wage rate $w_0 \neq 3$. Moreover, the subset $A_{2,w_0=3}$ clearly contains the best (more skilled and more motivated) workers so that, when the wage rate increases, worse workers will necessarily enter the market: for $w_0 > 3$ average productivity and average motivation will fall. Finally, note that condition (3) is not verified for $w_0 = 3$, $\mathrm{P}(B_{1,w_0=3})$ being clearly lower than $(1-\mathrm{P}(B_{1,w_0=3}))$.

Let's now consider Case B which is characterized by $\mathrm{E}[\theta|w]$ and $\mathrm{E}[\gamma|w]$ increasing in the wage rate (see Figure 4, Case B). In Figure 6, again, we observe the whole population inside the set of potential workers, together with the average values of θ and γ, the regression line and the line of marginal workers defined by a wage rate equal to 3.5. The line of marginal workers splits the set of active workers into the two subsets $B_{1,w_0=3.5}$ and $B_{2,w_0=3.5}$. As Figure 6 shows, condition (3) should be verified for $w_0 = 3.5$, $\mathrm{P}(B_{1,w_0=3.5})$ being clearly larger than $(1-\mathrm{P}(B_{1,w_0=3.5}))$. As before, monotonicity of $\mathrm{E}[\theta|w]$ and $\mathrm{E}[\gamma|w]$ in Figure 4 (Case B) implies that condition (2) is true for all other values of the wage rate $w_0 \neq 3.5$. Moreover, observing workers who are out of the vocational market for $w_0 = 3.5$, it is evident that, when the wage rate increases, better workers will enter the market (workers with higher productivity and higher motivation with respect to the average characteristics of those already active for $w_0 = 3.5$). Finally, note

that condition (2) is not verified for $w_0 = 3.5$, $\mathrm{P}(A_{2,w_0=3.5})$ being close to zero and so clearly lower than $(1-\mathrm{P}(A_{2,w_0=3.5}))$.

Cases C and B of the simulations can be interpreted following the very same reasoning.

Interestingly, and in line with Example 1, we observe that an important part of the information on the joint probability distribution function $f(\theta, \gamma)$ which is relevant for conditions (2) and (3) is summarized by the slope of the regression line. In particular, the regression line contains the information provided by the left hand side of the two conditions but it does not contain information on the gap between conditional and marginal means of the distribution as expressed by the right hand sides of the two conditions. In other words, the regression line does not allow us to take into account where conditional means $\mathrm{E}[\theta|A_{i,w_0}]$ and $\mathrm{E}[\gamma|B_{i,w_0}]$, with $i = 1, 2$, are located with respect to marginal means.

In Example 1, since the correlation was perfect, a positive covariance was sufficient to assure that one counterintuitive result occurred; the comparison between $r'(\theta)$ and b defined which one of the two effects occurred. On the contrary, for a general distribution function, a positive covariance and the comparison between $r'(\theta)$ and b are no longer sufficient and we only obtain *necessary conditions*.[15] Thus, we can state the following corollary:

Remark 5 *Assume a linear relationship between θ and γ is described by a linear dependence in mean, leading to $\gamma = a + b\theta + \tilde{\varepsilon}$, in which $\tilde{\varepsilon}$ is an error term with zero mean and $b = \frac{cov(\theta,\gamma)}{var(\theta)}$. When $cov(\theta, \gamma) > 0$, then one of two counterintuitive results may occur: average productivity of active workers can be decreasing in the wage rate for at least a sub-interval of possible salary levels when $r'(\theta) < b$, average vocation of active workers can be increasing in the wage rate for at least a sub-interval of possible salary levels when $r'(\theta) > b$.*

Intuitively, the necessary conditions illustrated in Remark 5 are likely to be satisfied if the conditional variance of γ, is small. In fact, by reducing the conditional variance of γ, we get closer and closer to the case of perfect correlation illustrated in Example 1 so that the conditions in Corollary 5 are also sufficient.

In line with Remark 3, in the Appendix 7.3 we provide some evidence that productivity and motivation are characterized by a positive conditional dependence ($cov(\theta, \gamma\,|w_0\,) > 0$) using data provided by the Italian survey ICSI 2007.

In particular, we consider the employees of the Italian cooperatives analyzed in the survey, that is, workers that entered the vocational market at a given salary. The ICSI survey includes the question "How do you define your employer-employee relationship with the cooperative?", the answer to which can be considered as a proxy for workers' motivation. Moreover, as a proxy for the workers' productivity,

[15] For example given $cov(\theta, \gamma) > 0$ and $r'(\theta) < b$, the counterintuitive effects with respect to θ can occur. However, if $\mathrm{E}[\theta|A_{2,w_0}]$ is very close to μ_θ, condition (2) is not verified.

we consider bonuses and cash prizes that they receive as an extra earning with respect to their monthly wage. It is worth noting that we do not interpret the association between motivation and productivity as causal, but simply as documenting their statistical connection.

Our empirical results in Appendix 7.3 are coherent with the literature on public administration showing that public service motivation is positively correlated with job performance in the public sector (see Petrovsky 2009, Naff and Crum 1999, Park and Rainey 2008, Ritz 2009 and Steijn 2008). Finally, Freeman (1997) finds evidence of positive dependence between productivity and vocation. The author considers volunteer workers, or the ones who are willing to work for nothing. In our model they are the ones with either a very high motivation or a very low outside option, or both. He shows that volunteers are indeed workers with high productivity also characterized by a high opportunity-cost to engage in the working-for-nothing activity. This suggests that many potential workers with characteristics close to $(\bar{\theta}, \bar{\gamma})$ exist, so that a positive dependence between productivity and vocation may result at least for high-productivity levels.

5.2 Policy implications, an example

A real example showing the policy relevance of our results concludes this part of the paper.

Suppose that, given the current wage rate w_0, the market is characterized by a shortage of workers (as is the market for nurses in many countries[16]). Our paper shows that a wage increase as a policy to deal with the shortage has some potential drawbacks. Indeed, when condition in inequality (2) holds, the overall quality of active workers deteriorates, since higher salaries attract less skilled and less motivated workers. On the contrary, when condition (3) holds, the pool of active workers will improve since higher salaries attract more skilled and more motivated workers. Both phenomena only occur when a positive correlation between skills and vocation exists and, in Appendix 7.3, we show through real data that such a positive correlation is plausible. Moreover, we are able to easily distinguish between the two opposite situations in the case of a linear dependence in mean between θ and γ. When a linear dependence exists, then by comparing the relative magnitude of the slope of the regression line and the slope of the curve of marginal workers, we can distinguish between the desirable situation and the undesirable one. In particular, a low positive correlation and a high incentives salary scheme in the alternative sector, that is, a high $r'(\theta)$, are likely to correspond to an improvement in the overall quality of the active workers.[17]

[16] See, among others, Antonazzo et al. 2003; Shields 2004; Simoens et al. 2005.

[17] The statistical relationship between motivation and productivity in the case of potential nurses could be calculated using data collected, for example, from students attending nursing schools.

5.3 The Lemons Problem and the counterintuitive effect on average productivity of active workers

We now consider the impact that the counterintuitive effect on the average productivity of active workers has on the inefficiency caused by adverse selection in the vocation based market. We noted in Remark 1 that average productivity of active workers is always increasing in the wage rate in the standard model. In the vocation-based labor market, on the other hand, if condition (2) in Proposition 1 holds, then the positive effect on average productivity of active workers generated by a positive variation in the wage rate is reversed, for at least a sub-interval of possible wage levels.

Corollary 1 in Section 3 states that the inefficiency due to adverse selection is mitigated by intrinsic motivation since, given a specific level of the wage rate, more productive workers enter the vocation-based market than the standard one. Vocation is thus beneficial since, all else being equal, it increases both the employment level and the average productivity of active workers. Putting together Corollary 1 and Proposition 1, we can conclude that:

Corollary 2 *In the vocation-based market and for every value of the wage rate, intrinsic motivation leads to an overall increase in the average productivity of active workers with respect to the standard model. If condition (2) holds, then the increase in the average productivity of active workers in the vocational market is mitigated but still persists.*

The monotonically decreasing curve in Figure 2 describes the average productivity of active workers when condition (2) holds for every value of the wage rate as in cases A and D of the simulations (depicted in Figure 4). Interestingly, Corollary 2 states that such a curve always lies above the increasing curve of average productivity of active workers in the standard model (i.e. the dashed curve in Figure 2).

6 Conclusion

We analyze adverse selection in a vocation-based labor market where both productivity and motivation are workers' private information and the wage offered by firms is uniform. We show that intrinsic motivation alleviates the Lemons Problem since average productivity of active workers increases compared to a standard non-vocational labor market. More interestingly, we analyze how the pool of active workers changes with the wage rate. In particular, we prove that the association properties between skills and motivation in the population of potential workers have a dramatic impact on the composition of the labor force and determine the characteristics of labor supply.

With respect to the question stated in the introduction, that is, "do higher salaries attract more productive but less motivated workers?", we show that the answer is positive only in one of the three possible scenarios that can be observed, that is in the *intuitive* one. Another two counterintuitive cases

are possible and occur only if a positive association exists between skills and motivation. The worst case scenario occurs when average vocation and average productivity of active workers simultaneously decrease as the wage rate increases. This case proves that the potential drawbacks of a wage increase in a vocation-based market are particularly serious: a wage increase can attract the worse workers, those with skills and abilities below average. The more desirable outcome, on the other hand, occurs when average vocation and average productivity of active workers are both increasing in the wage rate. Here a wage increase attracts better workers.

We show that the best outcome is likely when a positive but mild correlation characterizes the distribution of abilities and motivation in the population of potential candidates and the workers' outside option is sufficiently steep.

Our findings have important policy implications. A shortage in the labor market for nurses is documented in almost all developed countries (Antonazzo et al., 2003; Shields, 2004; Simoens et al., 2005), and a wage increase has been indicated as the most obvious policy measure. Our results provide some insights into the possible consequences of a wage increase on the quality of the new pool of active nurses.

A related debate is currently going on in Italy about the relative efficacy and desirability of the remuneration scheme designed for Members of Parliament: citizens (and some political parties) are asking for a reduction in the overall parliamentary wage. In fact, today, entering Parliament is a highly lucrative activity in Italy given that the real parliamentary wage has been growing at an average annual rate of 3.9% since 1980; however the median ability score of representative elected after 1994 has fallen and is today negative for all the major political parties (see Merlo et al. 2009). This phenomenon could be interpreted as a counterintuitive effect on the average productivity of active workers. In addition, our results indicate that also average motivation in the current pool of Italian politicians may be worse than before. Given the evolution of the remuneration scheme for elected representatives in the past years and the resulting change in their median ability score, our model suggests that a reduction in the parliamentary wage could be indeed an appropriate policy measure.

Concluding, we show that a positive or negative variation in the wage rate, as a policy measure to raise workers' quality may have unexpected and undesired effects on the composition of the active work-force and should be supported by some knowledge on the association properties of skills and vocation in the population of potential workers.

7 Appendix

7.1 Proof of Observation 1

Given w_0, the marginal workers' productivity in the standard non-vocational market is $\hat{\theta} = r^{-1}(w_0)$. For the same wage, the productivity level of marginal workers in the vocation-based market is $\tilde{\theta} = r^{-1}(w_0+\tilde{\gamma})$. We can compare productivity of the marginal workers in the two markets. Since the function $r(\cdot)$ is strictly increasing and $\tilde{\gamma} \geq 0$, it is $r^{-1}(w_0 + \tilde{\gamma}) \geq r^{-1}(w_0)$. Thus $\tilde{\theta} = \hat{\theta}$ for $\tilde{\gamma} = 0$ and, $\tilde{\theta} > \hat{\theta}$ for $\tilde{\gamma} > 0$. In other words: for every strictly positive $\tilde{\gamma}$ and $r\left(\underline{\theta}\right) < w_0 < r\left(\bar{\theta}\right)$, in the model where vocation matters marginal workers have higher productivity than in the standard model.

We now compare average productivity of active workers in the vocation-based market and in the standard non-vocational market. Let's consider the case where $\bar{\theta} > r^{-1}(w_0 + \bar{\gamma})$ or $\bar{\theta} > \theta_{\text{sup}}$ as in Figure 1, where θ_{sup} is the highest productivity level of workers accepting the job for a given salary w_0. The same reasoning can be applied when $\bar{\theta} < r^{-1}(w_0 + \bar{\gamma})$ or $\bar{\theta} = \theta_{\text{sup}}$.

The probability that workers enter the standard non-vocational market at a given salary w_0, is:

$$A = \int_{\underline{\gamma}}^{\bar{\gamma}} \int_{\underline{\theta}}^{r^{-1}(w_0)} f\left(\theta, \gamma\right) d\theta d\gamma,$$

whereas the probability that workers enter the vocation-based market conditional on w_0 is $A + B$, where:

$$B = \int_{\underline{\gamma}}^{\bar{\gamma}} \int_{r^{-1}(w_0)}^{r^{-1}(w_0+\gamma)} f\left(\theta, \gamma\right) d\theta d\gamma.$$

In particular, the expected value of θ given the salary w_0 in the standard non-vocational market, is:

$$E_{SM}\left[\theta \left| w_0 \right.\right] = \frac{\int_{\underline{\gamma}}^{\bar{\gamma}} \left[\int_{\underline{\theta}}^{r^{-1}(w_0)} \theta f\left(\theta, \gamma\right) d\theta\right] d\gamma}{A} = \frac{A'}{A}$$

and the expected value of θ in the vocation-based market given w_0 is

$$\begin{aligned} E_{VM}\left[\theta \left| w_0 \right.\right] &= \frac{\int_{\underline{\gamma}}^{\bar{\gamma}} \left[\int_{\underline{\theta}}^{r^{-1}(w_0)} \theta f\left(\theta, \gamma\right) d\theta\right] d\gamma + \int_{\underline{\gamma}}^{\bar{\gamma}} \left[\int_{r^{-1}(w_0)}^{r^{-1}(w_0+\gamma)} \theta f\left(\theta, \gamma\right) d\theta\right] d\gamma}{A + B} \\ &= \frac{A' + B'}{A + B} \end{aligned}$$

We now prove that $E_{VM}\left[\theta \left| w_0 \right.\right] \geq E_{SM}\left[\theta \left| w_0 \right.\right] \forall w_0$, or:

$$\frac{A' + B'}{A + B} \geq \frac{A'}{A}.$$

The previous condition can be rewritten as follows:

$$\frac{B'}{B} \geq \frac{A'}{A}. \tag{6}$$

The ratio $\frac{B'}{B}$ is the expected value of θ in the interval $\left(r^{-1}(w_0), r^{-1}(w_0 + \bar{\gamma})\right]$, whereas $\frac{A'}{A}$ is the expected value of θ in $\left(\underline{\theta}, r^{-1}(w_0)\right]$. The two expected values lie respectively in the two intervals that

are not overlapping and then $\frac{B'}{B} \in \left(r^{-1}(w_0), r^{-1}(w_0+\bar{\gamma})\right]$ and $\frac{A'}{A} \in \left(\underline{\theta}, r^{-1}(w_0)\right]$. Inequality (6) is thus always valid, for any given w_0, provided that the probabilities A and B are different from zero.

7.2 Proof of Remark 1

(i) Marginal workers in the non-vocational sector are $\hat{\theta} : r(\hat{\theta}) - w = 0$. By totally differentiating the previous equation with respect to $\hat{\theta}$ and w: $r'(\hat{\theta})d\theta - dw = 0$. Since the function $r(\cdot)$ is increasing, the first claim is obtained. Obviously, the higher the slope of the outside option function $r(\theta)$, the lower the impact of a wage increase on the productivity level of marginal types. (ii) From the proof of Observation 1, the average productivity of active workers in the non-vocational sector when the wage is w_0 can be written as follows:

$$E_{SM}\left[\theta\,|w_0\right] = \frac{\int_{\underline{\gamma}}^{\bar{\gamma}} \left[\int_{\underline{\theta}}^{r^{-1}(w_0)} \theta f\left(\theta,\gamma\right) d\theta\right] d\gamma}{\int_{\underline{\gamma}}^{\bar{\gamma}} \int_{\underline{\theta}}^{r^{-1}(w_0)} f\left(\theta,\gamma\right) d\theta d\gamma}$$

Since $h\left(\theta\right) = \int_{\underline{\gamma}}^{\bar{\gamma}} f\left(\theta,\gamma\right) d\gamma$, we can write:

$$E_{SM}\left[\theta\,|w_0\right] = \frac{\int_{\underline{\theta}}^{r^{-1}(w_0)} \theta h\left(\theta\right) d\theta}{\int_{\underline{\theta}}^{r^{-1}(w_0)} h\left(\theta\right) d\theta}$$

We now calculate the derivative of $E_{SM}\left[\theta\,|w_0\right]$ with respect to the wage rate and we show that it is always increasing:

$$\frac{\partial}{\partial w_0} E_{SM}\left[\theta\,|w_0\right] = \frac{h(r^{-1}(w_0))r^{-1}(w_0)\frac{\partial r^{-1}(w_0)}{\partial w_0}\int_{\underline{\theta}}^{r^{-1}(w_0)} h(\theta)d\theta - h(r^{-1}(w_0))\frac{\partial r^{-1}(w_0)}{\partial w_0}\int_{\underline{\theta}}^{r^{-1}(w_0)} \theta h(\theta)d\theta}{\left[\int_{\underline{\theta}}^{r^{-1}(w_0)} h(\theta)d\theta\right]^2}$$

The sign of $\frac{\partial}{\partial w_0} E_{SM}\left[\theta\,|w_0\right]$ has the same sign of the numerator (N) of the previous expression. N can be rewritten as:

$$N = h(r^{-1}(w_0))\frac{\partial r^{-1}(w_0)}{\partial w_0}\left[\int_{\underline{\theta}}^{r^{-1}(w_0)} \left[r^{-1}(w_0)h\left(\theta\right) - \theta h\left(\theta\right)\right] d\theta\right]$$

Since the function $r(\cdot)$ is increasing and $\theta \in \left[\underline{\theta}, r^{-1}\left(w_0\right)\right]$, N is always non-negative.

7.3 Dependences between productivity and vocation in real data

In Remark 3 we argued that Conditions (2) and (3) together suggest that the covariance between θ and γ conditional on the wage rate w_0 is likely to be positive.

To see whether our Remark is statistically relevant, that is to verify whether a positive dependence between productivity and motivation given the wage rate is plausible in the real world, we use data from the Survey ICSI 2007 (Indagine Cooperative Sociali Italiane, or the Survey on Italian Social Cooperatives).

Note that, by analyzing workers already in the vocation-based market (employees in the cooperatives) we are just verifying the sign of the conditional covariance defined by the wage rate observed in the market when the survey was performed: $cov(\theta, \gamma\,|w_0)$.

The survey consists of 4,134 interviews with employees and 388 interviews with managers, from 441 Italian cooperatives in the nonprofit sector. It is worth noting that the nonprofit sector can be reasonably considered as a vocation-based market. The survey comprises a large set of questions, ranging from socio-demographic controls (age, gender, education, etc.) to economic variable (e.g. wage), job characteristics (tasks, working hours, overtime) and job satisfaction with respect to a number of possible domains (with colleagues, wage, type of job).

In particular the ICSI survey includes the question "How do you define your relationship with the cooperative?", the answer to which can be considered as a proxy for workers' motivation. In fact, workers were asked to give their degree of consensus (on a 1 to 7 scale) to the following possible answers to the previous question:

1. a mere contractual relationship where a job is exchanged for pay.
2. a contribution which helps the cooperative to reach its goal.
3. a mix between professional growth and personal development.
4. a set of relationships which goes beyond a mere contractual relationship.
5. a social commitment shared by the respondent and the cooperative.

The previous statements are indicated in our empirical analysis, respectively, as Vocation_1, Vocation_2, Vocation_3, Vocation_4, Vocation_5, which are dummy variables.

As a proxy for the productivity, we consider bonuses and cash prizes that workers received as an extra earning with respect to their monthly wage. Furthermore, the monetary value of monthly benefits possibly received by employees (for instance free phone calls) is added to bonuses and cash prizes. Of course, while all workers provided an answer to the questions concerning intrinsic motivation, only a fraction of them declared that they received monthly bonuses and cash prizes or monthly benefits, or both. In particular, the percentage of workers that received benefits or bonuses is about 28% of the total workers. For this reason, to measure the relation between motivation and productivity we consider a Tobit model, in which "monthly bonuses and cash prizes with benefits" ($bonus\_benefit$) is the dependent variable. We consider the following specifications:

$$bonus\_benefit = \begin{cases} \beta' X + \epsilon & \text{if } bonus\_benefit > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

in which ϵ is a Gaussian error term with zero mean, X is the vector of regressors, and the linear part

of the model is defined as

$$\begin{aligned}\beta' X \quad = \quad & \beta_0 + \beta_1 sex + \beta_2 italian + \beta_3 permanent + \beta_4 status + \beta_5 type + \\ & \beta_6 age + \beta_7 no\_school + \beta_8 primary\_school + \beta_9 \sec ondary\_school + \\ & \beta_{10} professional\_school + \beta_{11} other\_school + \beta_{12} high\_school + \\ & \beta_{13} univ\_degree + \beta_{14} south + \beta_{15} north/west + \beta_{16} north/east + \\ & \beta_{17} tenure + \beta_{18} full\_time + \beta_{19} worked\_hours + \delta_i vocation\_i.\end{aligned}$$

In particular we evaluated 5 different models, in correspondence of the five vocation statements, i.e., vocation_1,. . . ,vocation_5.

We also considered a number of dummy variables as control variates. In particular, *permanent* means that the worker has a permanent position, *status* specifies whether the worker is also a member of the cooperative or not, *type* refers to the type of the cooperative[18], *south*, *north/west*, *north/east* and *center* refer to the geographical location of the cooperative[19], *tenure* indicates the years spent in the cooperative by the worker, $full\_time$ indicates full time job, and finally $worked\_hours$ states the hours worked per month in the cooperative. We also considered the nationality (*italian*) and the schooling level (no_school, $primary\_school$, $secondary\_school$, $professional\_school$, $high\_school$,$univ\_degree$, $univ\_laurea$ and $other\_school$)[20].

Our empirical findings are illustrated in Table 1. In the specification we find that all the vocational coefficients except the first one (which, as expected, is negative) have a positive and significant impact on productivity; in particular, consensus to Vocation_1 and Vocation_2 are significant at the 1% level, while consensus to Vocation_3, Vocation_4, and Vocation_5 are significant at 5%. These findings provide evidence of a positive dependence between vocation and productivity for a given level of wage, at least when productivity is measured by "monthly bonuses and cash prizes with benefits".[21]

Our empirical analysis suggests that the hypothesis of positive conditional correlation between voca-

[18] Two types of cooperatives are considered: "cooperatives A" manage health and education services, "cooperatives B" have the goal of inclusion of disadvantaged workers (disabled, ex-prisoners, ex-drug addicts...) in industry, agriculture and trade.

[19] Note that *center* has been omitted to avoid collinearity.

[20] In particular we indicate with $univ\_degree$ a 3 years University degree, whereas with $univ\_laurea$ we refer to a 4/5 years degree.

[21] A possible objection to the previous empirical strategy is that there could be a firm-specific effect in the cooperative sector, meaning that the majority of the cooperatives give almost all their workers some bonuses and/or benefits. However, as shown in Figure 7, the proportion of cooperatives providing to a given percentage of workers bonuses and/or benefits is substantially uniform over the sample. From the figure, it is clear that only a few cooperatives provide extra wages to all the workers, while many cooperatives provide extras only to a low fraction of people.

tion and productivity is reasonable.

Insert Table 1 here

Insert Figure 7 here

An interesting caveat of the empirical analysis is that we cannot use the workers' wage rate as a proxy for productivity, even if workers' monthly wage is available in the data (some small differences in the workers' wage rate exist, suggesting that the wage rate is not exactly uniform in this vocation-based sector). In fact, by considering the wage rate as a proxy for productivity we implicitly assume that the wage rate is always increasing in productivity, which obviously implies that productivity *is always* increasing in the wage. However, our results show that average productivity of active workers can be decreasing in the wage rate. Thus, if we used the wage rate as a proxy for productivity as in the standard empirical literature on labor, the empirical analysis would not be coherent with our theoretical results.[22]

7.4 Non uniform wage in the vocation-based market

We now consider the case where firms in the vocation-based market offer a wage that depends on the workers' productivity: $w = w(\theta)$, with $w'(\theta) \geq 0 \ \forall \theta \in \left[\underline{\theta}, \overline{\theta}\right]$. In other words skills are rewarded both in the vocation-based market and outside. We will derive conditions that allow us to extend results obtained considering the uniform wage to the present case of a salary that is increasing in productivity.

Workers' participation constraint becomes:

$$r(\theta) \leq w(\theta) + \gamma \tag{8}$$

We make the following additional assumptions:

- $r(\theta) \geq w(\theta) \quad \forall \theta \in \left[\underline{\theta}, \overline{\theta}\right]$. For every level of workers' productivity the salary is higher outside the vocation based market than inside. This assumption is documented by some empirical literature on the nonprofit negative wage gap (see, for example, Mervis and Hackett 1983, Weisbrod 1983 and Preston 1989) and on public-private wage differentials (for a recent contribution see Bargain and Mally 2008).[23] Obviously, both the nonprofit sector and the public one can be considered vocational-markets.

[22] Such an empirical strategy is precisely used in Becchetti *et al.* (2009) who use the same ICSI 2007 survey, and find clear evidence of positive correlation between intrinsic motivation and wage. In particular, in their empirical specification based on the *monthly wage* as dependent variable, the authors found that consensus to the last four vocational statements has a positive and significant effect on wages.

[23] Delfaauw and Dur's theoretical model (2010) also predicts the public-private wage gap. In particular, under the assumption that the demand for the public sector output is not too high, they find that the equilibrium price of public sector output must be lower than the equilibrium price of output in the private sector.

- $r'(\theta) > w'(\theta) \geq 0 \quad \forall\theta \in \left[\underline{\theta}, \overline{\theta}\right]$. For every level of workers' productivity incentives provided by the wage scheme are higher outside the vocation based market than inside.

Under the previous assumptions potential workers face a trade-off that is very similar to the one we saw before for high-productivity workers (see Section 3): by accepting the job in the vocation-based market, workers can benefit from their vocation γ but they earn a lower salary than if they choose the outside-option.

The curve of marginal workers is now $\gamma(\theta) = r(\theta) - w(\theta)$, where $\gamma(\theta)$ is a non-negative and increasing function $\forall\theta \in \left[\underline{\theta}, \overline{\theta}\right]$. Proposition 1 still holds given a wage function $w(\theta)$ which satisfies the previous assumptions. To see why, we propose the following example where the wage schedules $r(\theta)$ and $w(\theta)$ are both linear.

We still assume that the vocational market is small with respect to the alternative one, so that we can keep $r(\theta)$ constant when the salary offered in the vocation-based sector increases.

Example 2 *Let's consider that* $r(\theta) = \alpha\theta$, $\forall\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, *where* $\alpha > 1$. *Let's also assume that* $w_k(\theta) = \theta + \beta_k$, $\forall\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, *where* β_k *is a positive number such that* $k = 0, 1, 2, ...n$ *and* $\beta_0 = 0 < \beta_1 < \beta_2 < ... < \beta_n$. *Moreover,* β_n *is the wage increase such that worker* $\left(\bar{\theta}, \underline{\gamma}\right)$, *that is the one with the highest net reservation wage, is indifferent between entering and not entering the vocational market:* $W_{\max} = \bar{\theta} + \beta_n = \alpha\bar{\theta} - \underline{\gamma}$ *or* $\beta_n = (\alpha - 1)\bar{\theta} - \underline{\gamma}$. *Thus,* $\gamma(\theta) = r(\theta) - w_k(\theta) = (\alpha - 1)\theta - \beta_k \geq 0$. *Moreover,* $\gamma'(\theta) = (\alpha - 1) > 0$ *is the slope of the curve of marginal workers.*

From progressively adding β_1,β_2,...,β_k,...,β_n *to the initial wage schedule* $w_0(\theta) = \theta + \beta_0 = \theta$, *the curve of marginal workers shifts south-east maintaining its shape and its slope exactly as in the previous case with a uniform wage. Thus, Proposition 1 still holds. In particular, given a wage rate* $w_k(\theta) = \theta + \beta_k$, *we can identify the two areas of active workers (see Definition 3) in the following way:* $A_{1,w_k(\theta)}$ *is the set of workers entering the market with productivity below the average, i.e., such that* $\theta \leq E[\theta] = \mu_\theta$; *whereas* $A_{2,w_k(\theta)}$ *is the set of workers entering the market with productivity above the average, i.e.,* $\theta \geq E[\theta] = \mu_\theta$. *Thus:* $A_{1,w_k(\theta)} \cup A_{2,w_k(\theta)}$ *is still the set of workers entering the market at the salary* $w_k(\theta)$ *with* $E[\theta|A_{2,w_k(\theta)}] > \mu_\theta$ *and* $E[\theta|A_{1,w_k(\theta)}] < \mu_\theta$.

Referring to the case of a linear dependence in mean between productivity and vocation we can extend Remark 5 to the case of a non-uniform wage:

Remark 6 *Suppose that firms in the vocational sector offer a non-uniform wage* $w(\theta)$ *and that the relationship between the two variables* θ *and* γ *is described by a linear dependence in mean such that* $\gamma = a + b\theta + \tilde{\varepsilon}$, *where* $\tilde{\varepsilon}$ *is an error term with zero mean and* $b = \frac{cov(\theta,\gamma)}{var(\theta)}$. *When* $cov(\theta, \gamma) > 0$, *then for at least a sub-interval of possible salary levels, one of two counterintuitive results can occur: average productivity of active workers can be decreasing in the wage rate when* $r'(\theta) - w'(\theta) < b$, *average vocation of active workers can be increasing in the wage rate when* $r'(\theta) - w'(\theta) > b$.

Intuitively, when workers receive a non-linear wage also in the vocational sector, the curve of marginal workers must properly take into account the interaction between the total impact of incentives inside and outside the vocational market and the characteristics of the joint distribution function (summarized by the regression line). Of course, in the Example before, the comparison is between the slope of the regression line and $(\alpha - 1)$.

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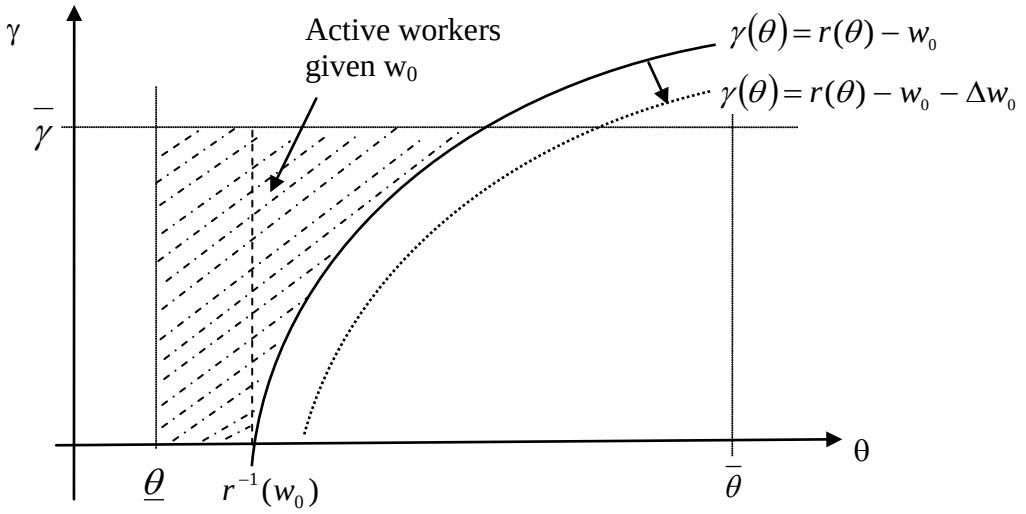

Figure 1: the set of potential workers and the curve defining marginal workers given salary w_0.

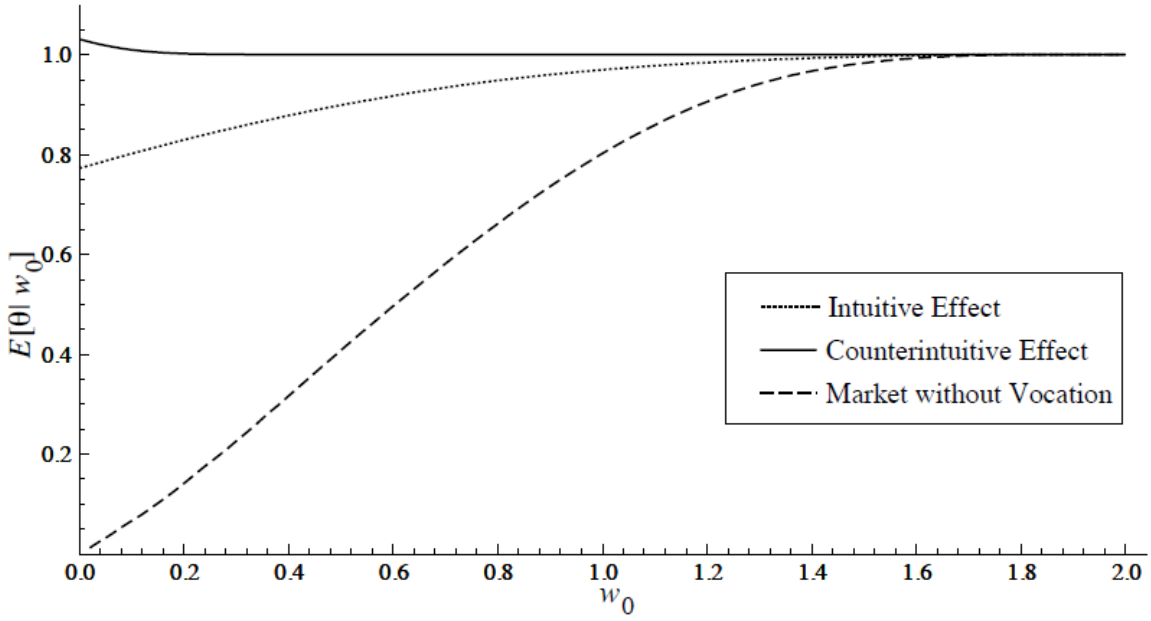

Figure 2: expected productivity given the wage rate. In the standard market it is monotonically increasing in the wage rate. In the vocational market, instead, counterintuitive effects can occur. In the figure, a case with average productivity of active workers monotonically decreasing and a case with average productivity monotonically increasing in the wage rate are shown.

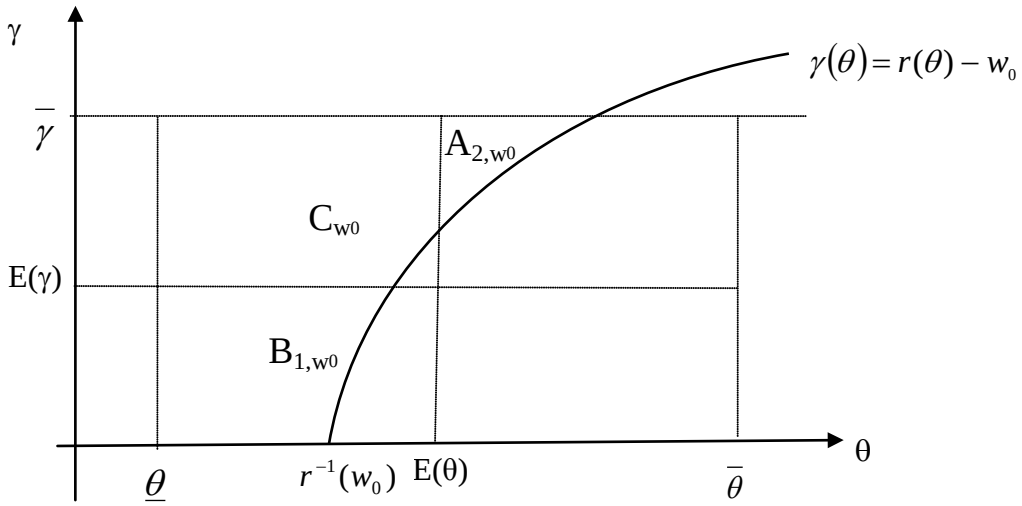

Figure 3: illustration of Remark 3. Decomposition of the covariance between θ and γ conditional on the wage rate w_0. Conditions (2) and (3) are compatible with a positive conditional covariance.

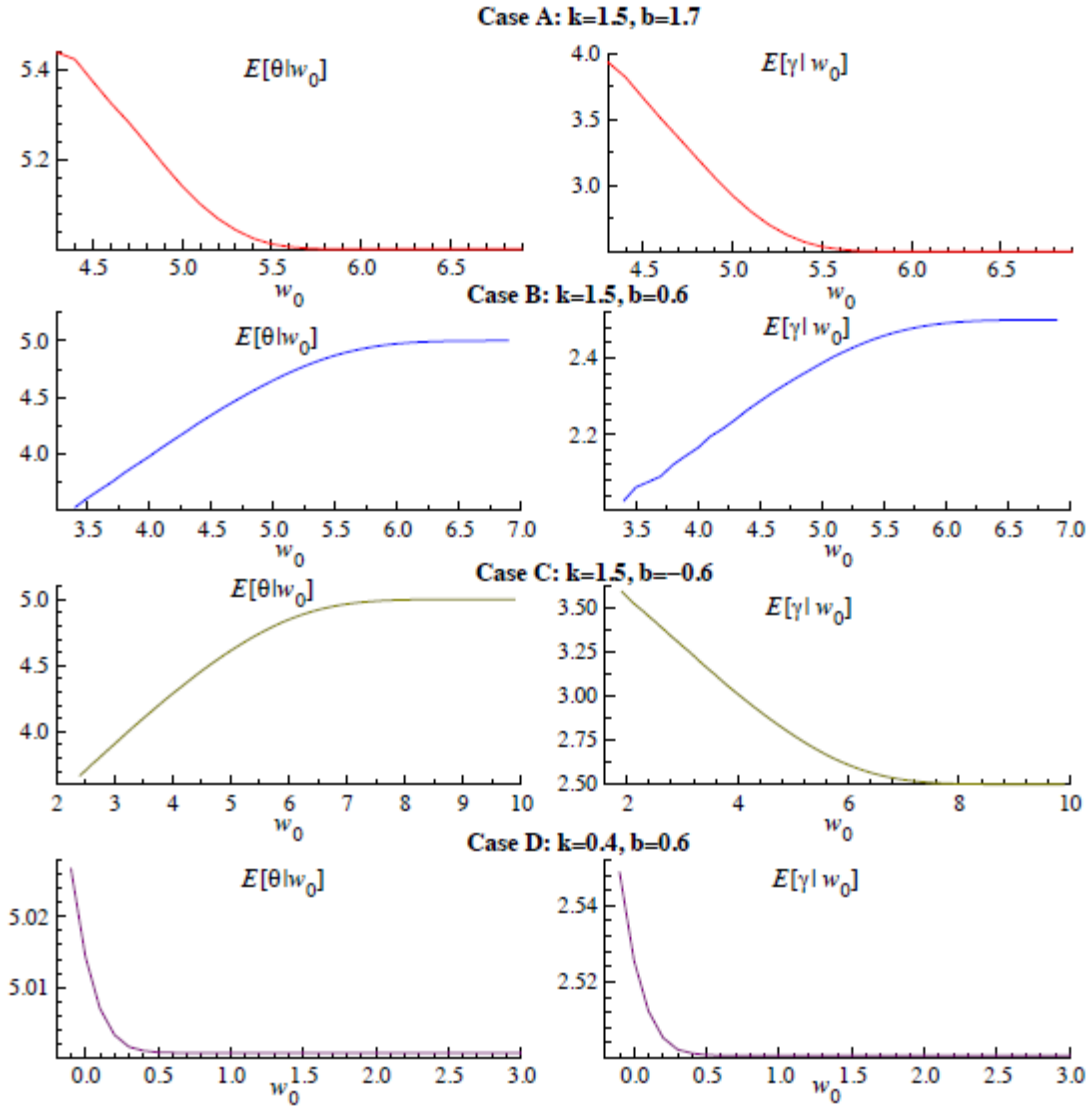

Figure 4: Simulations provided in Section 5. Cases A, B and D show counterintuitive effects. Case C show intuitive effects. Cases A and D present the counterintuitive effects as for average productivity of active workers, Case B as for average vocation.

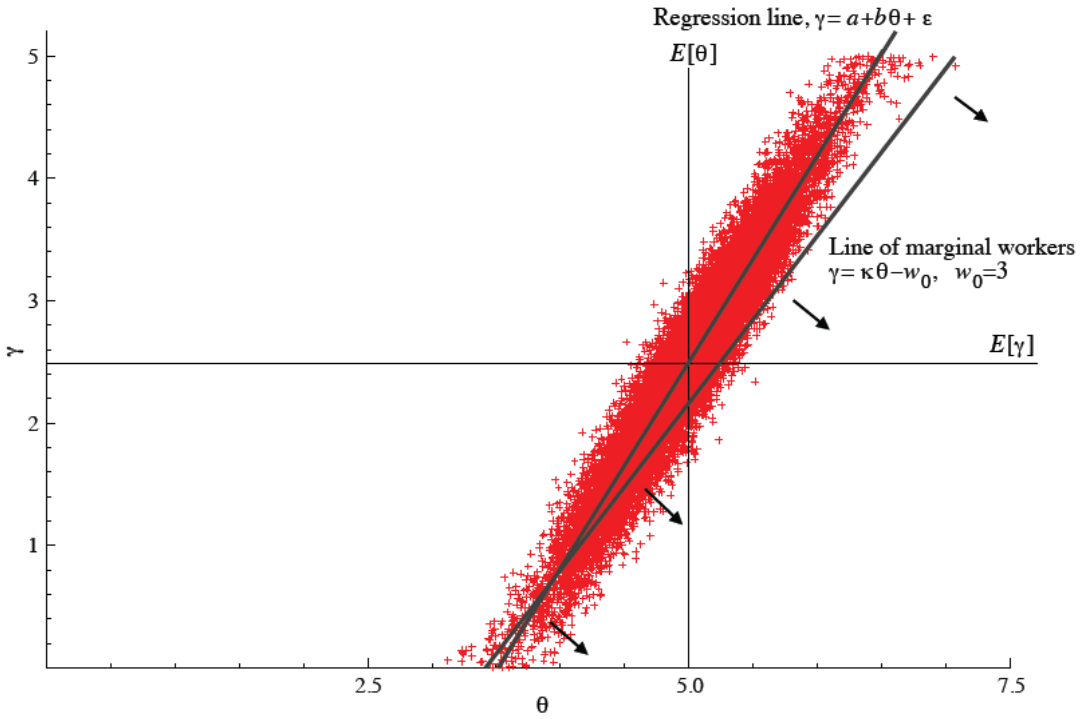

Figure 5: Explanation of case A of the simulations.

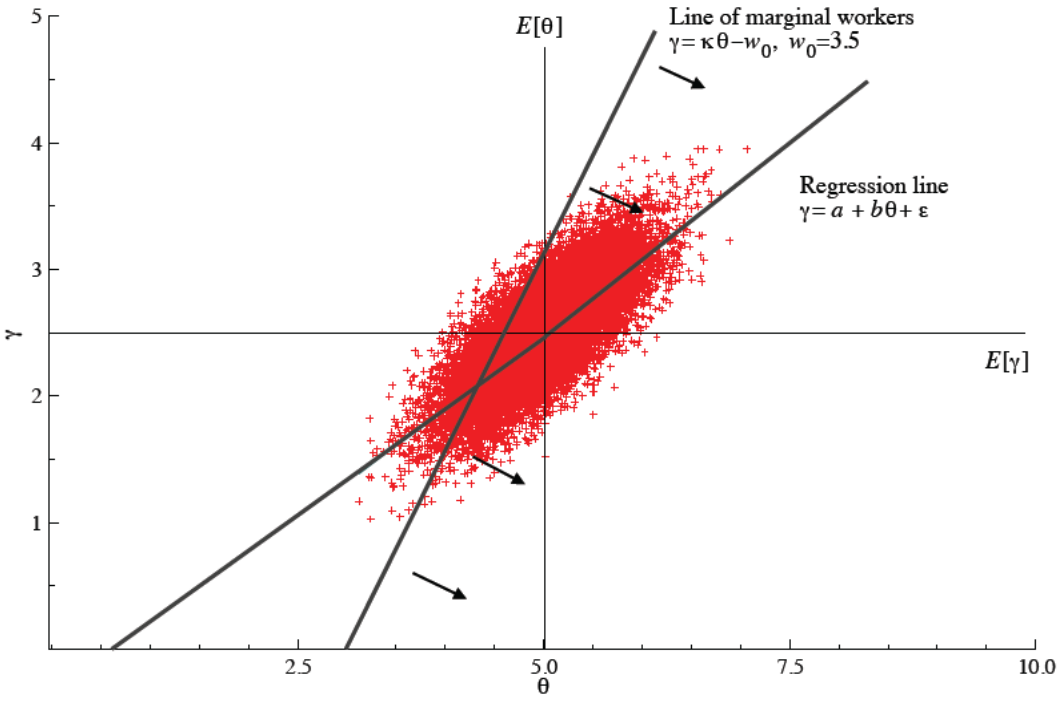

Figure 6: Explanation of case B of the simulations.

Number of cooperatives for each category.

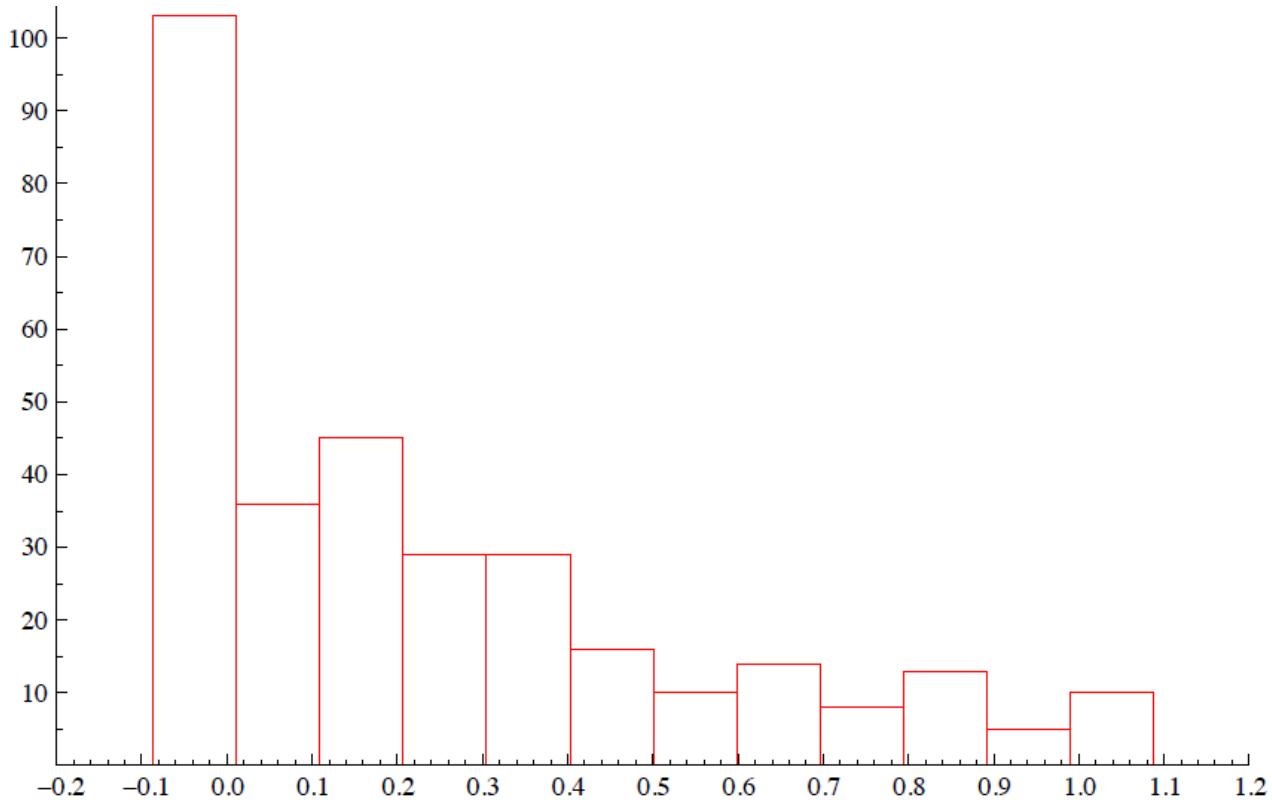

Figure 7: Histogram of frequencies for cooperatives that provide bonus/benefits to their workers. In particular, in the x-axis we report the percentage of workers for each cooperative receiving bonuses and/or benefits in the sample (Survey ICSI 2007).

TABLE 1. Tobit Model. *Dependent variable:* ***BENEFITS PLUS BONUSES*** **** means significant at 1%, ** at 5% and * at 10%.*

const	-1860.95	***	-2257.65	***	-2226.93	***	-2184.51	***	-2175.13	***
sex	135.618	***	128.568	**	127.931	**	123.644	**	128.128	**
italian	15.3855		45.4654		40.8144		44.8461		44.3500	
permanent	462.559	***	461.957	***	469.063	***	464.393	***	459.487	***
status	378.882	***	381.179	***	385.729	***	388.054	***	386.788	***
A type	-431.714	***	-413.712	***	-434.242	***	-421.656	***	-427.582	***
age	-0.533432		-0.656533		-0.195184		-0.711416		-0.999248	
no school	-23.156		-54.4346		-40.0614		-6.94014		-38.0849	
primary school	-326.979	**	-401.120	**	-399.083	**	-399.782	**	-394.588	**
secondary school	-277.421	***	-303.338	***	-307.822	***	-308.494	***	-314.514	***
professional school	-296.157	***	-324.635	***	-322.786	***	-322.210	***	-329.232	***
other school	-170.878	**	-180.945	***	-183.255	***	-182.048	***	-188.584	***
high school	-562.378		-638.001		-597.138		-588.568		-628.079	
univ diploma (3yrs)	-97.4013		-115.973		-113.089		-111.007		-117.497	
sud	-206.937	**	-209.015	**	-199.466	**	-209.830	**	-204.551	**
north/west	316.320	***	318.187	***	321.079	***	322.294	***	326.532	***
north/east	289.248	***	289.737	***	296.056	***	299.542	***	305.393	***
tenure	29.8192	***	29.0749	***	30.3267	***	30.2453	***	30.2334	***
full time	129.238	*	126.275	*	131.140	*	123.459	*	128.840	*
worked hours	19.8909	***	19.7434	***	19.7742	***	20.0376	***	19.9401	***
vocation 1	-56.1404	***								
vocation 2			47.4571	***						
vocation 3					35.7160	**				
vocation 4							31.8308	**		
vocation 5									31.4378	**